COVID-19 effects on the Canadian term structure of interest rates

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Abstract

In Canada, COVID-19 pandemic triggered exceptional monetary policy interventions by the central bank, which in March 2020 made multiple unscheduled cuts to its target rate. The aim of this paper is to assess the extent to which Bank of Canada interventions affected the determinants of the yield curve. By applying Functional Principal Component Analysis to the term structure of interest rates we find that, during the pandemic, the long-run dependence of level and slope components of the yield curve is unchanged with respect to previous months, although the shape of the mean yield curve completely changed after target rate cuts. Bank of Canada was effective in lowering the whole yield curve and correcting the inverted hump of previous months, but it was not able to reduce the exposure to already existing long-run risks.

JEL Classification: E43, E58, G01.

Keywords: Canadian yield curve, COVID-19, monetary policy, Functional Principal Components Analysis, smoothing.

1 Introduction and motivation

On[1] March 11, 2020, the director of the World Health Organization, Dr. T.A. Ghebreyesus, declared: *"We have therefore made the assessment that COVID-19 can be characterized as a*

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1 Abbreviations: PCA (Principal Component Analysis); FPCA (Functional Principal Component Analysis); FDA (Functional Data Analysis).

pandemic. [...] All countries must strike a fine balance between protecting health, minimizing economic and social disruption, and respecting human rights." (WHO Director-General's opening remarks at the media briefing on COVID-19, March 11, 2020).

This address officially marked the start of a health crisis that was already wreaking havoc on the global economy and financial markets. At the social level, on September 28, 2020 the world reached the terrible milestone of one million deaths due to COVID-19, and this death toll keeps growing (3.45 millions as of May 23, 2021; data from WHO COVID-19 Weekly Epidemiological Update of May 25, 2021). At the economic level, global economy saw an unprecedented collapse due to the pandemic. The Canadian gross domestic product fell by 7.1% and 11.4% during the periods from February to March 2020 and from March to April 2020, respectively (Statistics Canada, Table 36-10-0434-01). No sector of the economy has been spared from the crisis. The COVID-19 outbreak rapidly increased financial market volatility and augmented investors' fear. As to the Canadian bond market, these phenomena induced a widening of credit spreads, a liquidity shrinkage and a fall of bond funds value (Ouellet Leblanc and Shotlander, 2020).

In response to these events, the Bank of Canada announced several measures to reduce panic and calm down the markets. The intervention with the most powerful impact on bond yields was the huge cut of the overnight rate target (the target rate) from 1.75% to 0.25% in March 2020. This interest rate, which is employed by depository institutions in the overnight market, is the lowest possible interest rate and it has an extremely short-term nature. The central bank introduced several other measures to reduce the financial distress and promote the resilience of the Canadian economy, such as the Contingent Term Repo Facility, the Commercial Paper Purchase Program and the Corporate Bond Purchase Program, among the others.[2]

The yield curve (or term structure of interest rates) represents government bonds yields as a function of their time to maturity (or term). It is apparent that the monetary policy intervention of the Bank of Canada affects short-term yields. However, the impact on long-term yields is not straightforward. On the one hand, the mechanic compounding of interest rates makes the change in the target rate propagating across time horizons. On the other hand, the economic literature acknowledges that the long-term side of the term structure is generally mainly associated with expectations and anticipation of market participants about future macroeconomic scenarios. Detailed discussions of the expectation theory and its alternatives can be retrieved in Russell (1992) and in the introductions of Cox, Ingersoll, and Ross (1985) and Severino (2021).

Our goal is to assess the extent to which the Bank of Canada interventions affected

[2]Details on the Bank of Canada website at links: Contingent Term Repo Facility, Commercial Paper Purchase Program and Corporate Bond Purchase Program.

the determinants of the term structure of bond yields during the first wave of COVID-19 pandemic. We tackle the issue of studying the behavior of the yield curve across different time periods, using a data set of Canadian government bond yields ranging from May 1, 2018 to October 30, 2020, and we analyze these data employing a non-parametric approach within the framework of Functional Data Analysis. This allows us to quantify the evolution of the main factors of the yield curve over time, and to relate the changes in such factors to both market uncertainty and monetary policy interventions. Functional Data Analysis represents a very fruitful statistical tool to study the term structure of yields, since it permits to treat these data as curves and to fully exploit the shape information they comprise. In the realm of time series, some applications of FDA techniques are provided by Cai, Fan, and Yao (2000) and Ramsay and Ramsey (2002). On a larger perspective, an application of FDA to COVID-19 epidemic data can be found in Boschi et al. (2020).

In this paper, we provide a comparative study on three sub-periods of the data set (the latter including COVID-19 outbreak). We employ depth-based functional boxplots to visualize the curve distribution in each period, and apply Functional Principal Component Analysis (Ramsay and Silverman, 2005, Chapters 8 and 9) to each sample of yield curves in order to elicit the components explaining the most variability in each period. This FPCA of yield curves extends to the functional setting the more common application of the classic (non-functional) Principal Component Analysis to yields (Litterman and Scheinkman, 1991), which aims at decomposing the yields at different horizons into three factors: *level*, *slope* and *curvature*. Similarly to Litterman and Scheinkman (1991), we find three components reflecting the modes of variation of yields in the short-, medium- or long-term, that are not directly observable in the average or standard deviation yield curves. An important advantage of using FPCA instead of PCA is that the time dependence and the ordering of terms in the yield curve are implicitly taken into account in the functional analysis, while PCA treats the variables independently of their order. In addition, the interpretation of the functional components is neater, due to the smoothness of the component eigenfunctions, as well as of the means and the covariance functions. An example of application of FPCA to financial data (in particular, to volatility) is represented by Müller, Sen, and Stadtmüller (2011).

The FDA analysis is complemented by the estimation of Nelson and Siegel (1987) exponential regression model for the yield curve in each day – which allows us to describe the three factors from a different angle. In particular, the curvature effect, which has a tiny weight in the FPCA variance decomposition, is better appreciated in this model.

Our analyses show that Bank of Canada target rate cuts of March 2020 induced a decline in the entire term structure of yields and imposed a positive monotony to the yield curve (that was absent in the previous twelve months). Nonetheless, the FPCA reveals the non-negligible presence, during the pandemic, of the same long-run risks detected in the previous months. Central bank interventions had little impact on them. All conclusions are summarized in Section 6.

The paper is organized as follows. After describing the financial perspectives in Section 2, we illustrate the sample, its subdivision in periods and the related descriptive statistics in Section 3. In Section 4 we apply FPCA to the three sample periods. Section 5 completes the analysis by fitting Nelson and Siegel (1987) model. The main findings are, then, elaborated in Section 6. The Appendix contains further analyses and figures.

2 Financial perspectives and literature

The identification of the determinants of the yield curve and the forecast of their evolution constitute important challenges in financial economics. The yield curve reflects the health of the economic system and, when it is downward sloping, it is considered a preliminary signal for a financial crisis (Ang, Piazzesi, and Wei, 2006).

Several studies aim at understanding the main factors of the yield curve. In their seminal work, Litterman and Scheinkman (1991) use PCA to reduce the dimensionality of yields and identify three uncorrelated factors (namely, level, slope and curvature) that explain more than 98% of US Treasury bonds yields variance (see also Jamshidian and Zhu, 1996). The level represents a downward or upward change in interest rates characterized by a parallel shift in the yield curve. The slope (or steepness) reflects a twist caused by long-term rates being higher than short-term rates, or vice versa. Finally, a shock in the curvature generates an increase of both short- and long-term rates, and a simultaneous fall of intermediate rates, or vice versa (a butterfly). Importantly, the uncorrelation between the three components is crucial for the design of factor neutrality models to immunize portfolios from movements in such factors. See, e.g., Barber and Copper (1996), Falkenstein and Hanweck (1997) and Golub and Tilman (1997). The applications to Asset Liability Management by insurance companies and pension plans to hedge interest rate risks are countless. See, for instance, Chapter 7 in Veronesi (2016) and the comprehensive exposition in Luckner et al. (2003). Other uses of PCA in the yield curve modeling can be found in Novosyolov and Satchkov (2008) and in Barber and Copper (2012) and a recent application to European sovereign bonds is provided by O'Sullivan and Papavassiliou (2020).

In the literature, numerous three-factor models for the yield curve, alternative to the PCA approach, have been proposed (see, e.g. Balduzzi et al., 1996; Dai and Singleton, 2000). Recently, Ortu, Severino, Tamoni, and Tebaldi (2020) use persistence-based factors to model the term structure of interest rates and improve the predictability of bond returns.

Another branch of this literature focuses more on the identification of observable factors for the yield curve modeling (see, e.g., Evans and Marshall, 2007; Ang and Piazzesi, 2003; Bikbov and Chernov, 2010). In a nutshell, some macroeconomic variables (inflation, real activity...) largely contribute to explain movements in the yield curve at short, medium or long maturities.

Our paper aims at understanding the evolution of the factors of the yield curve in the COVID-19 period, in order to clarify the effects of the pandemic (and the consequent expansionary monetary policies) on bond markets. The loss of trust in the financial system rapidly led to massive corporate bond sales. As Kargar et al. (2020) illustrate, the severe illiquidity in US corporate bond markets in March 2020 was mitigated only by several Federal Reserve interventions (the liquidity crisis microstructure is described by O'Hara and Zhou, 2021). Similar dynamics have been recognized in investment funds in corporate bond markets by Falato, Goldstein, and Hortaçsu (2020). Only Federal Reserve announcements were able to calm down the disruption in the debt market (Haddad, Moreira, and Muir, 2021). The illiquidity of US Treasury bonds during COVID-19 outbreak has also been analyzed by He, Nagel, and Song (2021) in a dynamic equilibrium framework. Specifically, our paper focuses on the changes in the three main determinants of the yield curve (level, slope and curvature) in the Canadian government bond market.

3 Sample subdivision and descriptive analysis

We consider a daily sample of yields from May 1, 2018 to October 30, 2020. The data set can be freely downloaded from the Bank of Canada website at `https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/`. For each day, the data set contains the yields to maturity of zero-coupon bonds with term ranging from 3 months to 30 years, on a quarter basis (120 maturities in total). That is, the data set consists of daily yield curves generated from pricing data of Canadian government bonds and treasury bills. A description of the methodology used to derive the yield curves is provided in Bolder, Metzler, and Johnson (2004).

3.1 Sample subdivision

Since we aim at quantifying the effects of the COVID-19 outbreak on the term structure of rates, we partition our data set into three sub-samples, based on the occurrence of important financial events. It should be observed that the target rate is 1.25 % on May 1, 2018, when our sample begins, and is later modified several times by the Bank of Canada (see Table 1 and top panel of Figure 1).

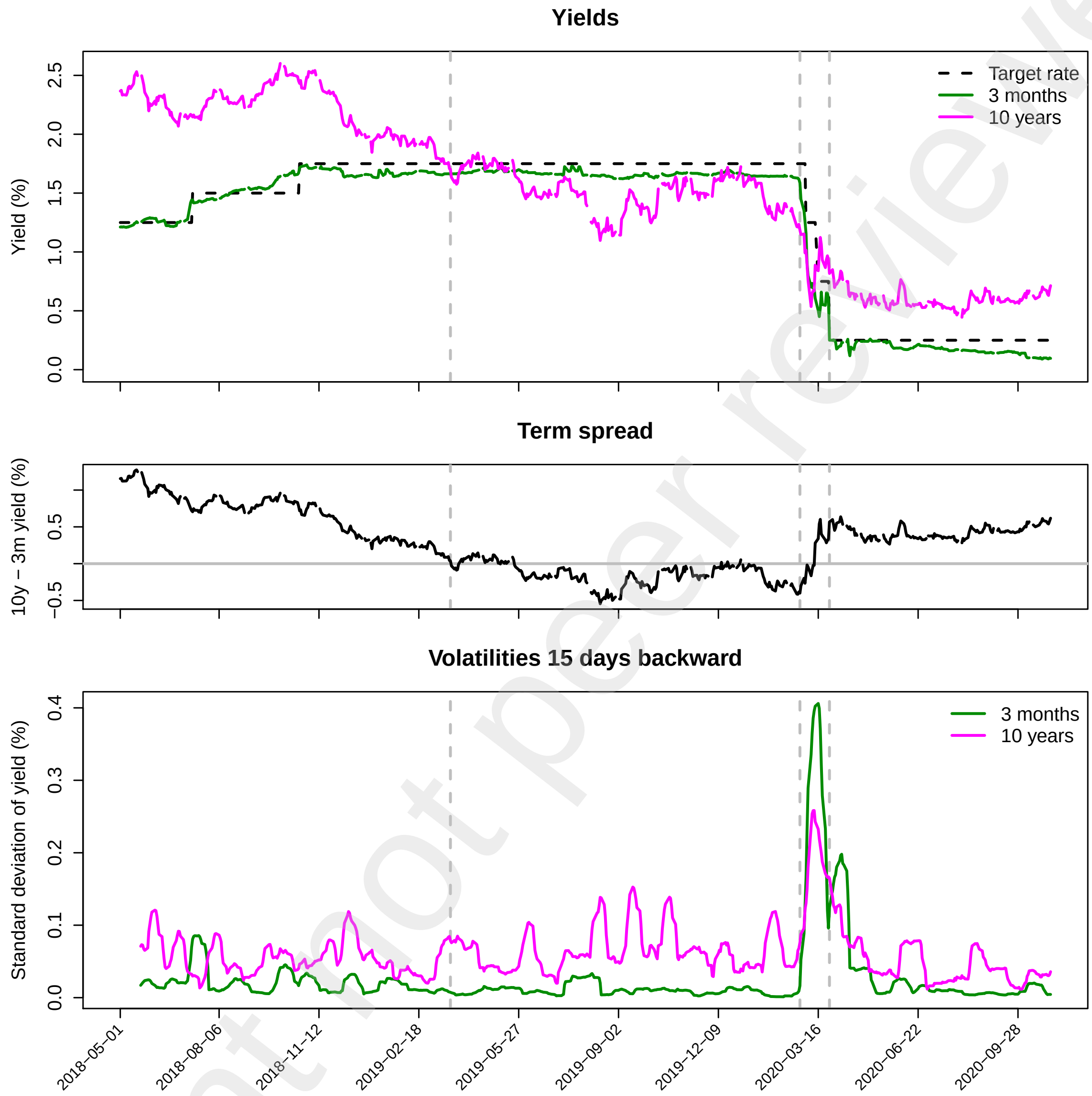

Figure 1: **Daily yield data from May 1, 2018 to October 30, 2020.** The top panel shows the target rate set by the central bank (black dashed line), the 3-month yield to maturity (green line), and the 10-year yield to maturity (purple line). The middle panel displays the term spread (10-year yield minus 3-month yield). The bottom panel represents the 15-day backward standard deviations of 3-month and 10-year yields (green and purple lines, respectively). The vertical gray dashed lines show our subdivision in periods. All values are in percentages.

Table 1: Target rates set by Bank of Canada in the sample period. The rate of May 1, 2018 was fixed previously. Dates in dd/mm/yy format.

Date	01/05/18	10/07/18	24/10/18	04/03/20	16/03/20	27/03/20
Target rate	1.25 %	1.50 %	1.75 %	1.25 %	0.75 %	0.25 %

The first sample split results from the comparison of the 3-month and 10-year yields (see the top and middle panels in Figure 1). The latter is higher at the beginning of the sample, as it is in normal times, but becomes lower later. Hence, we set the beginning of the second period to the first day in which the term spread (the difference between 10-year and 3-month yields) is zero, i.e. to March 22, 2019. The second period is characterized by a negative (or close to zero) term spread.

The second period ends before the pandemic outbreak. Although the first rate cut occurs on March 4, 2020, a pre-announcement effect is observable: the 3-month yield remarkably falls on February 28, 2020, and the yield curve features a consistent downward shift. Hence, we choose this date as the cut-off. Our subdivision is supported by functional boxplots based on the 2-curve band depth (Sun and Genton, 2011; López-Pintado and Romo, 2009). Indeed, including February 28 to March 3, 2020 (a total of three curves) in the second period would have produced three outliers in the corresponding functional boxplot. The selected split excludes the presence of outliers (see Figure A1).

We remove from the analysis the period from February 28 to March 27, 2020, due to the exceptionally frequent interventions by the Bank of Canada in cutting the target rate and starting its support to the economy. This phase features a high demand for liquidity in the markets, pervasive uncertainty and financial stress. The yield curves in this removed transition period do not feature a common pattern and largely vary from one day to the other (see Figure A1). Moreover, yield volatility is exceptionally high in this period, especially in the short term (see the bottom panel of Figure 1).

From March 28, 2020, no other cuts of target rates take place, the term spread is stable and volatility is under control. This is our third period of analysis, the one related to the pandemic.

As a result, the sub-samples under scrutiny are the following.

- *First period:* from May 1, 2018 to March 21, 2019; 233 observations. The term spread is positive (normal time); the Bank of Canada increases the target rate twice; some tensions are linked to the trade war between China and the United States, but global economic growth keeps solid (in particular in the United States).

- *Second period:* from March 22, 2019 to February 27, 2020; 245 observations. The term spread is negative, on average (anomalous time); the Bank of Canada does not intervene on the target rate; somber economic perspectives anticipated by market actors; slowdown of global economic growth, in particular in China and the Euro area (because of Brexit concerns); trade conflicts between China and the United States are still present.

- *Third period (COVID-19):* from March 28, 2020 to October 30, 2020; 124 observations. The term spread is positive (return to normal time); the Bank of Canada does not intervene on the rates anymore, but renovates its economic support; sharp contraction of the world economy due to COVID-19 first wave (massive reduction of economic activity to limit the virus spread).

More detailed macroeconomic issues are described in the quarterly Monetary Policy Reports by the Bank of Canada, from April 2018 to October 2020.

3.2 Mean and volatility term structure

For each day, we consider the term structure of interest rates, i.e., the curve of bond yields to maturity at different horizons. We then estimate, for each of the three periods considered, the mean yield curve and the relative volatility (coefficient of variation, i.e., standard deviation divided by the mean) curve. Figure 2 shows these curves after smoothing according to the preliminary steps of the FPCA algorithm described in Subsection 4.1. For the sake of completeness, we plot in the same figure the raw curves: pointwise mean and relative volatility. Note that we consider relative volatility in place of volatility because yields variability is naturally lower when yields are low (as in the third period). This problem is overcome by the relative volatility.

We observe a neat decline of mean yields (at all horizons) from the first period to the second, and to the third. The decline from the second to the third period follows closely the Bank of Canada target rate cuts in March 2020. Interestingly, the first and the third periods show increasing yield curves, with the first period curve attaining a plateau after few maturities. On the contrary, the average yield curve for the second period features an inverted hump, with medium-term yields lower than short- and long-term ones.

As far as relative volatility is concerned, the third period shows an excess short-term volatility caused by the COVID-19 outbreak. The hump of relative volatility gives an idea of the horizon (roughly 10 years) after which the pandemic consequences on government bonds are expected to completely fade out (the peak is, however, roughly at 3-year maturity). In addition, the relative volatility at long maturities of the COVID-19 period is very similar

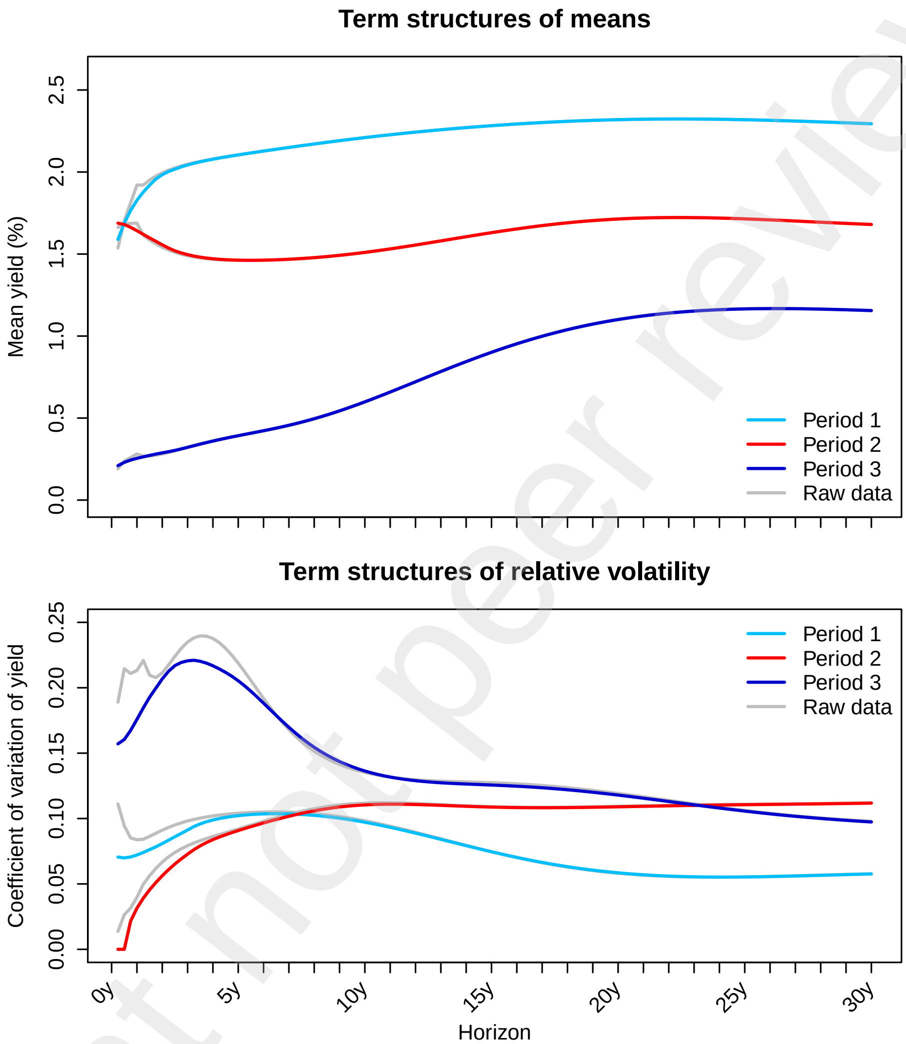

Figure 2: **Mean and relative volatility term structures.** Smoothed term structures of mean (top panel) and relative volatility (bottom panel) of yields to maturity with different horizons, estimated separately for each of the three periods. Light blue, red and blue lines correspond to the first, second and third period, respectively. Grey lines represent the raw curves.

to the one of the second period, suggesting the presence of long-run risks with similar magnitude. The central bank interventions do not seem to have affected such sources of uncertainty. Interestingly, the long-run relative volatility is smaller in the first period, where market tensions concentrate in the short and medium run.

To summarize, from mean and relative volatility curves in the three sub-samples, we can infer the following.

- *First period:* increasing mean yield curve with high yields; the curve reaches the plateau quickly; low long-run relative volatility.
- *Second period:* mean yield curve with inverted hump: medium-term yields lower than short- and long-term yields; average yields generally lower than the ones in the first period; sustained long-run relative volatility.
- *Third period (COVID-19):* increasing mean yield curve with low yields; the curve reaches the plateau later than the first period curve; explosion of short-term relative volatility; sustained long-run relative volatility as in the second period.

4 Functional Principal Component Analysis of yield curves

4.1 Methodology

We employ Functional Principal Component Analysis (FPCA, see e.g. Ramsay and Silverman, 2005, Chapters 8 and 9) to study the modes of variation of the yield curves in each of the three sub-samples. For each period we consider the yield curves $y_1(x), \dots, y_N(x)$ observed at horizons $x = 3, 6, \dots, 360$, as N realizations of the square-integrable stochastic process $Y(x)$, $x \in \mathcal{X} = [3; 360]$, with mean $\mu(x) = \mathbb{E}(Y(x))$ and covariance function $v(x, z) = Cov(Y(x)Y(z))$. We then project the random curve $Y(x)$ into the low dimensional space defined by the first K smooth eigenfunctions $\phi_1(x), \dots, \phi_K(x)$ of the covariance operator $V : L^2(\mathcal{X}) \to L^2(\mathcal{X})$, $V(f) = \int_{\mathcal{X}} v(x, z)f(x)dx$, corresponding to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$:

$$Y(x) \approx \mu(x) + \sum_{k=1}^{K} \xi_k \phi_k(x),$$

where $\xi_k = \int_{\mathcal{X}} [Y(x) - \mu(x)]\phi_k(x)dx$ are the principal component scores. To perform FPCA and estimate the mean, covariance function, eigenfunctions, eigenvalues, and principal component scores, we follow the PCA through Conditional Expectation (PACE) approach of Yao, Müller, and Wang (2005) and Liu and Müller (2009), as implemented in the function `FPCA` of the R package `fdapace` (Carroll et al., 2020). In order to incorporate smoothing in the FPCA, we employ local linear smoothing for estimating the mean $\mu(x)$ and the covariance function $v(x, z)$ by setting the argument `methodMuCovEst` to `"smooth"`. Briefly, the employed PACE approach involves the following steps:

- Compute the estimated mean function $\hat{\mu}(x)$ using local linear smoothing, aggregating the N curves $y_1(x), \ldots, y_N(x)$;
- Compute the estimated covariance function $\hat{v}(x, z)$ by smoothing the sample raw covariance;
- Obtain the estimated eigenfunctions $\hat{\phi}_k$ and eigenvalues $\hat{\lambda}_k$ by performing eigenanalysis on the smoothed covariance function;
- Estimate the principal component scores using numerical integration, i.e. compute $\hat{\xi}_{i,k} = \int_3^{360} [y_i(x) - \hat{\mu}(x)] \hat{\phi}_i(x) dx$.

In each period, we select $K = 3$, obtaining the approximation $y_i(x) \approx \hat{\mu}(x) + \sum_{k=1}^{3} \hat{\xi}_{i,k} \hat{\phi}_k(x)$ for each yield curve. This choice leads to a variability explained greater than 99% in all three periods, and matches the choice of three factors (the so-called *level*, *slope* and *curvature*) in PCA on the yield curve usually made in the literature (Litterman and Scheinkman, 1991).

FPCA results in the three periods are displayed in Figures 3-5, which include barplots of the variance explained by the first three components, the corresponding eigenfunctions, and the shocked yield curves when the shock affects a single component (i.e. the plot of the components as perturbations of the mean).

4.2 Results interpretation

In line with Litterman and Scheinkman (1991), the first three principal components can be interpreted as level, slope and curvature, and explain most of the variations in the yield curves: the variance explained is more than 99% in each period. In particular, the level alone explains always more than 93% of total variance, while the slope has more weight in the second period than in the other two periods.

For each of the three periods, the level (first component) is rather stable across horizons (top-right and bottom-left panels of Figures 3-5). However, the first period features a slightly stronger level in the medium term, a behavior similar to the relative volatility in the same period. In the other periods, the level is higher in the long term than in the short term and a shock to the level induces a vertical shift in the yield curve which is larger in the long term. In fact, the COVID-19 crisis did not meaningfully modify the main mode of variation (the level). Long-run risks are still present and their magnitude is unchanged,

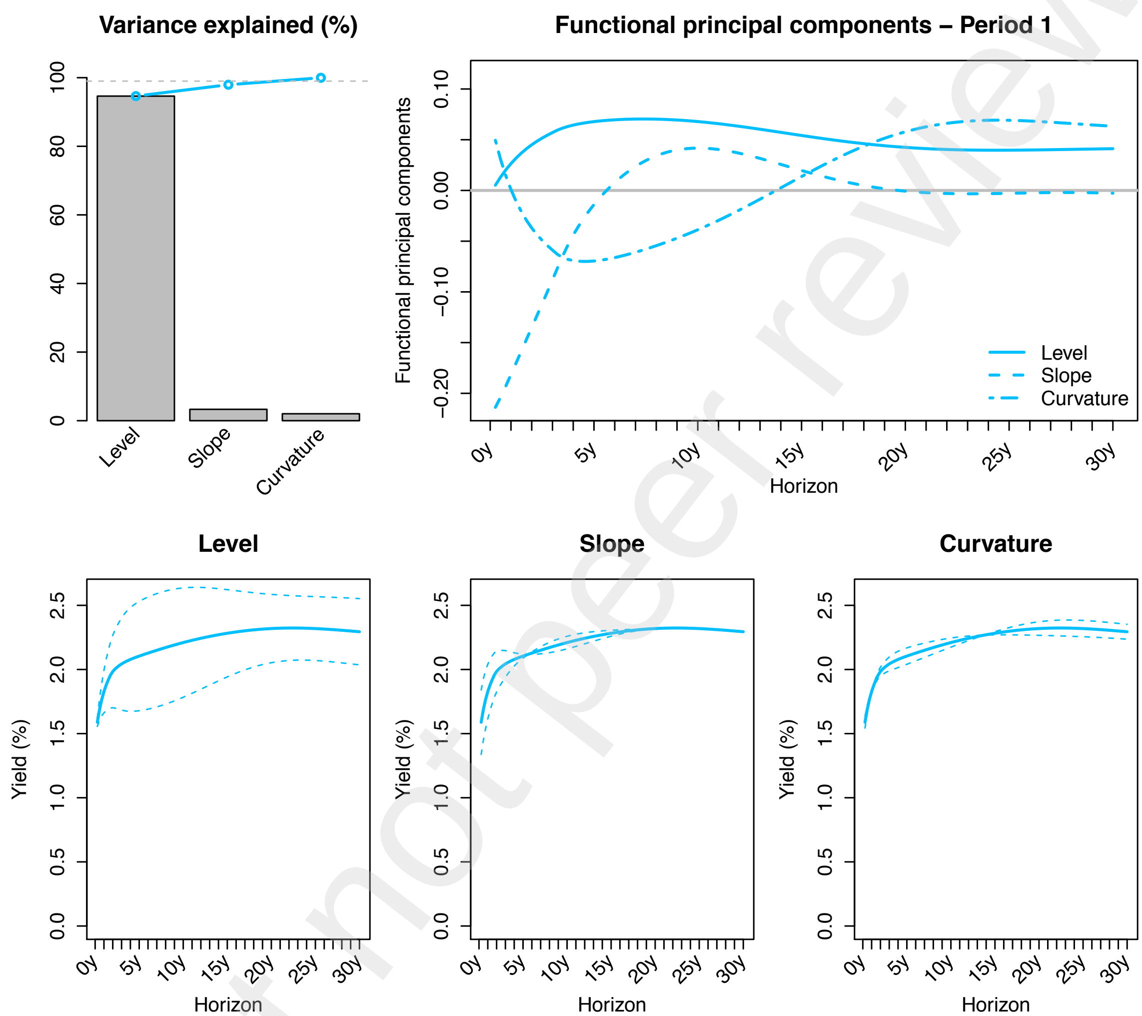

Figure 3: **FPCA results for the first period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the first period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

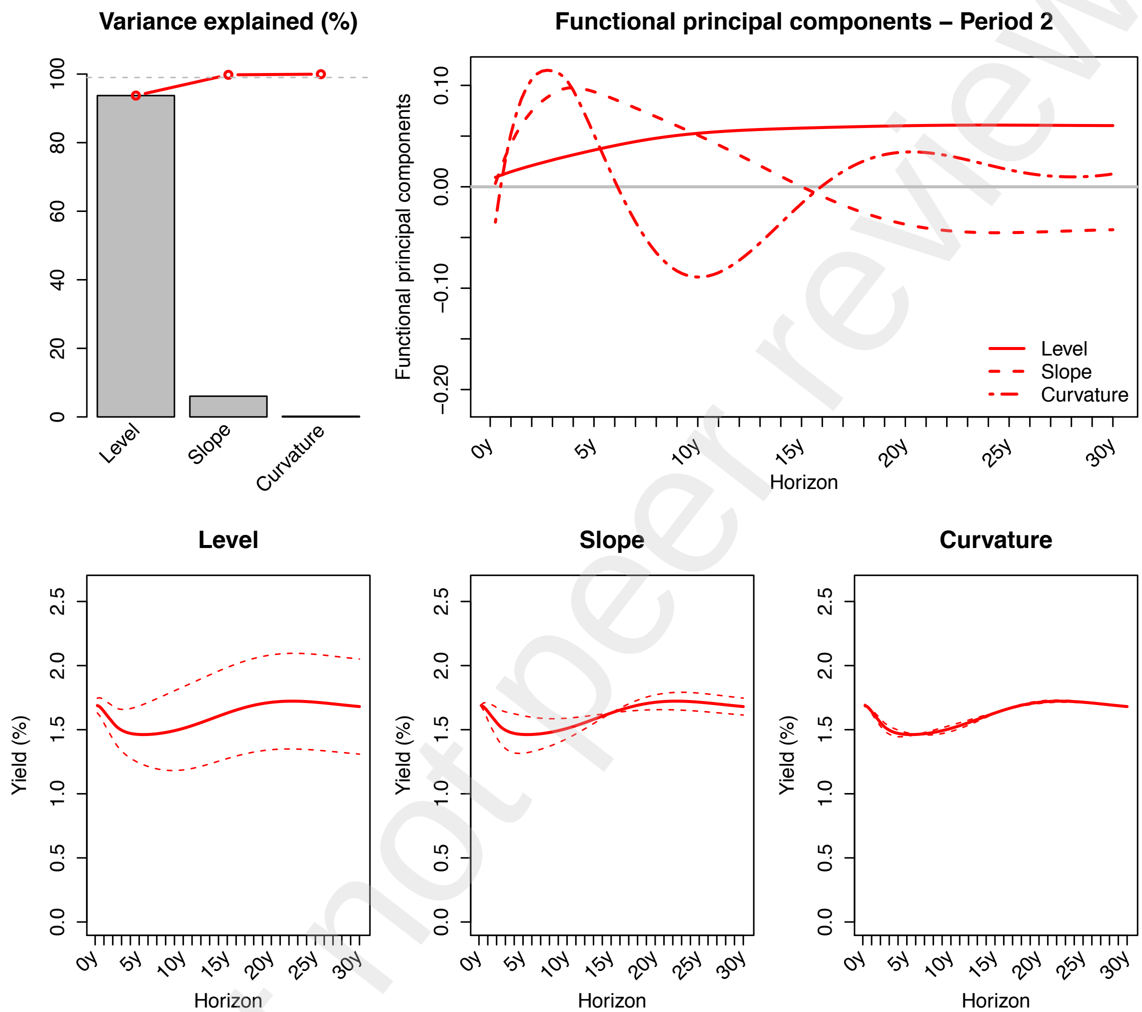

Figure 4: **FPCA results for the second period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the second period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

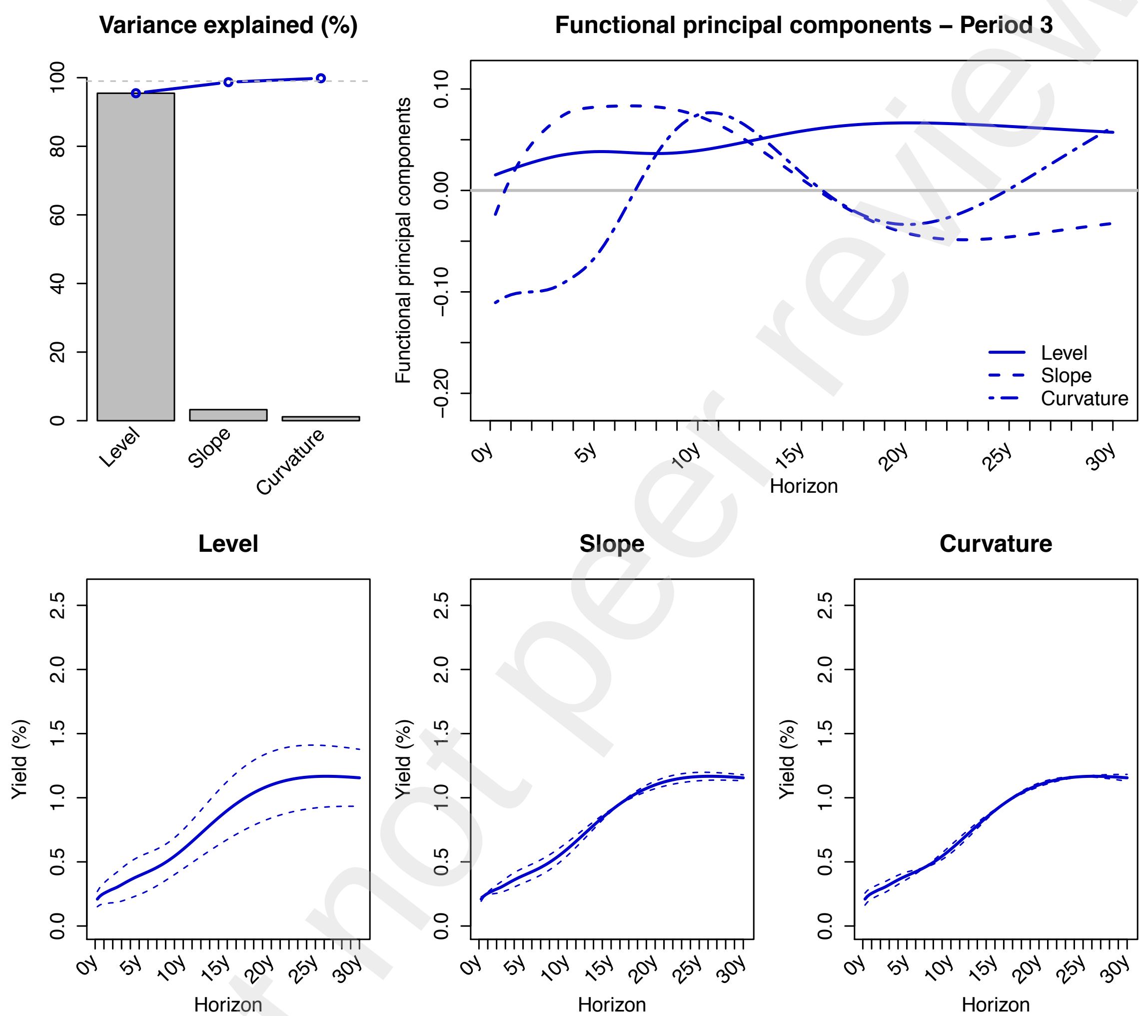

Figure 5: **FPCA results for the third period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the third period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

even though all yields to maturity are lower in the third period: interest rates continue being persistent.

The first period features a slope (second component) remarkably different from the other periods (top-right and bottom-center panels of Figures 3-5). In particular, in the first period the slope is high (in absolute terms) on maturities less than 5 years, smaller (and of opposite sign) between 5 and 17 years roughly, and becomes zero later. Such behavior is in line with the pattern of relative variance and the rapid flattening of the mean curve, and it reconfirms the importance of short- and medium-term risks in the first period. A positive shock in the slope can make short-term yield higher than medium-term ones without affecting long-term rates. On the contrary, the slope in the other periods is non-null in the long run (after the 15-year horizon roughly). Interestingly, the slope long-run dependence is qualitatively unchanged during the pandemic. The differences regard the variance explained (6% and 3.22% in the second and third periods, respectively) and the shape of the shocked yield curves. Indeed, in the second period, a positive shock in the slope can make the yield curve flat or even downward sloping (see the bottom-center plot in Figure 4). In the third period, a modest shock in the slope does not affect the increasing monotony of the yield curve.

In the first two periods, the eigenfunctions of third principal component (curvature) have the same signs in the short- and in the long-term. The behavior of the curvature is more complex in the third period. The interpretation is hard in this case. However, the variance explained by the curvature is tiny in all periods (roughly 2% at most).

5 Nelson and Siegel (1987) approach

Nelson and Siegel (1987) model provides an alternative perspective on level, slope and curvature effects. This model allows us to shed some light on the role of the curvature, which does not explain much variability in FPCA, making the interpretation difficult.

5.1 Methodology

Nelson and Siegel (1987) provide a parsimonious exponential approximation of the yield curve, based on three factors. As illustrated by Diebold and Li (2006), for each day t the yield curve can be estimated via the functional form

$$y_t(x) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t x}}{\lambda_t x}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t x}}{\lambda_t x} - e^{-\lambda_t x}\right),$$

where $y_t(x)$ is the yield to maturity with term x months and λ_t is a positive parameter that governs the exponential decay. Here, $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are three latent dynamic factors with long-term, short-term and medium-term effects. As discussed in Diebold and Li (2006),

$\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ can be interpreted as *level*, *slope* and *curvature* factors. Specifically, $\beta_{1,t}$ coincides with the long-term yield, $\beta_{2,t}$ is closely related with the term spread and $\beta_{3,t}$ is associated with twice the two-year yield minus the sum of the ten-year and three-month yields.

The decay parameter λ_t is chosen as to maximize the medium-term regressor when

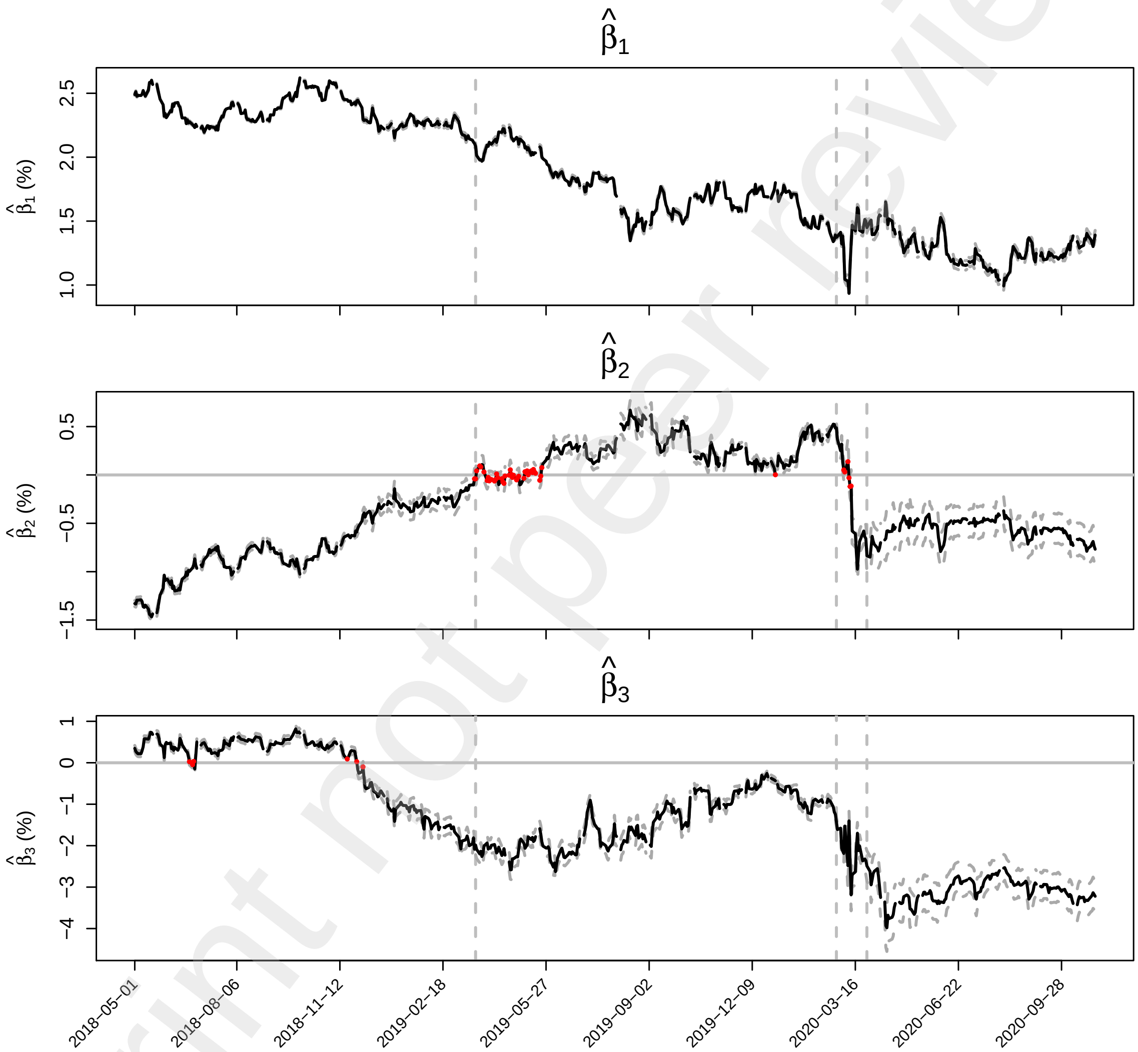

Figure 6: **Nelson and Siegel (1987) factors.** Time series of estimated $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ and their 95% confidence intervals in the Nelson and Siegel (1987) model (with $\lambda_t = 0.0609$). The vertical gray dashed lines show our subdivision in periods. All values are in percentage. Red points correspond to non-significant coefficients (p-value of t-test for $\beta_{j,t} = 0$ larger than 0.05, for $j = 1, 2, 3$). See also Figure A5.

$x = 30$ months. This approach leads to $\lambda_t = 0.0609$ (see Diebold and Li, 2006). The betas are then estimated via ordinary least squares for each day t. Figure 6 shows the obtained time series of daily beta estimates. Descriptive statistics for the betas in each of the three periods are collected in Table 2. See also Figure A5 for the corresponding t-test p-values.

Table 2: Descriptive statistics – mean, volatility (standard deviation) and relative volatility (coefficient of variation) – of Nelson and Siegel (1987) model in each of the three periods.

	Mean (%)	Volatility (%)	Relative volatility
Period 1			
$\hat{\beta}_{1,t}$	2.3542	0.1224	0.0520
$\hat{\beta}_{2,t}$	-0.7065	0.3532	0.4999
$\hat{\beta}_{3,t}$	-0.1449	0.8546	5.8982
Period 2			
$\hat{\beta}_{1,t}$	1.7471	0.2151	0.1231
$\hat{\beta}_{2,t}$	0.2435	0.1831	0.7521
$\hat{\beta}_{3,t}$	-1.4169	0.6153	0.4342
Period 3			
$\hat{\beta}_{1,t}$	1.2750	0.1339	0.1050
$\hat{\beta}_{2,t}$	-0.5466	0.0992	0.1814
$\hat{\beta}_{3,t}$	-3.0557	0.3131	0.1025

5.2 Results interpretation

The $\beta_{1,t}$ estimates decrease, on average, from the first to the second period, as well as from the second to the third one (Figure 6 and Table 2). Such decrease reflects the declines of mean long-term yields along the three periods we observed in the top panel of Figure 2. The Bank of Canada cuts of its target rate in March 2020 triggered the decrease of $\beta_{1,t}$ from the second to the third period. In addition, the relative volatility of $\hat{\beta}_{1,t}$ is remarkably high in the second and third periods. This behavior mirrors the one of the relative volatility at far maturities (Figure 2) and reflects the long-run dependence of the level detected by the FPCA. Interestingly, this behavior is not captured by the volatility itself.

The signs of the $\hat{\beta}_{2,t}$ reflect the yield curve inversion of the second period. Indeed, $\hat{\beta}_{2,t}$ is almost always positive in the second period, while it is negative in the other periods (Figure 6 and Table 2). The COVID-19 period features an increasing yield curve, similarly to the first period. The central bank interventions contributed to correct the curve inversion of the second period. In addition, the relative volatility of $\hat{\beta}_{2,t}$ is higher in the second period, mirroring the higher variance explained by the second functional principal component (slope) in the second period, with respect to the other periods.

The behavior of $\hat{\beta}_{3,t}$ permits to better understand the curvature in the three periods. As shown in Diebold and Li (2006), this factor is closely related with twice the two-year yield minus the sum of the ten-year and three-month yields. $\hat{\beta}_{3,t}$ decrease over the three period, on average, capturing a more and more valuable curvature effect. Indeed, in the second period, the curvature is due to the inverted hump in the mean yield curve. In the third period, although the yield curve is increasing, the mean $\hat{\beta}_{3,t}$ is low because of relatively high long-term rates with respect to the two-year yield.

6 Conclusions and further discussion

Our methodology relies on the application of FPCA to the term structure of yields to maturity and it is complemented by the (exponential) regression model of Nelson and Siegel (1987). A simple but key step of the analysis is the split of the sample period, which permits to detect and quantify the different behaviors of the yield curves. Future works involve the development of a time-dependent FPCA, which would permit us to follow the evolution of the components in a continuous way along the time, overcoming the issue of splitting the sample in separate periods. For comparison, the outcomes of (classic) PCA are displayed in Figures A2-A4. Results are qualitatively similar to FPCA, but they are less smooth, hence harder to interpret. These differences between FPCA and PCA would be more striking with more noisy and/or more sparsely sampled yield curves, and highlight the advantages of working in a functional framework.

Regarding the Nelson and Siegel (1987) approach, the presence of unit roots in the sequences of $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ could be inspected. However, we do not judge such analysis insightful for understanding the changes in the yield curve during the pandemic.

In summary, we can draw the following conclusions on the consequences of the Bank of Canada interventions on the term structure of Canadian government bond yields.

- During the COVID-19 pandemic, the average term structure of rates is increasing, as it was before March 21, 2019: Bank of Canada interventions contribute to cancel the curve inversion observable between March 22, 2019 and March 3, 2020. In addition, yields at all maturities in the COVID-19 period are lower than in previous periods. The target rate cut by the central bank induced a decrease of rates also at long maturities.

- During the COVID-19 crisis, relative volatility is extremely high for bond yields with close maturities. This reflects a high level of uncertainty about the near future. Moreover, the long-run relative variance is not different from the one of the twelve months before. Despite Bank of Canada stimuli, the same amount of long-run risk is present.

- The level (in FPCA) has the same long-run dependence during the pandemic and in the twelve months before. This provides evidence of the maintained persistence of yields to maturity despite their overall decrease.

- During the pandemic, the slope (in FPCA) keeps the same long-run dependence of the twelve months before. However, it explain less variance and a modest shock in the slope is not able to modify the monotony of the yield curve. Indeed, the increasing monotony of the yield curve induced by the target rate cuts is rather insensitive to shocks in the slope component.

- In the COVID-19 period, a curvature effect (from Nelson and Siegel, 1987, model) is present even though the yield curve does not display any hump. This is caused by relatively high long-term yields with respect to medium-term yields.

In a nutshell, the FPCA of pandemic is similar to the one of the twelve months before, while the mean yield curve is likely to be a downward shift of the mean curve dating back to the period before March 21, 2019. By modifying the target rate, the Bank of Canada was effective in lowering all yields to maturity and correcting the inverted hump in the curve, but it was not able to affect the (remarkable) long-run risks.

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A1 Appendix

Figure A1 shows the functional boxplots of the yield curves on the different time periods described in Subsection 3.1.

Figures A2-A4 contain the outcomes of the classic PCA (for comparison, FPCA results can be found in Figures 3-5 in the main text).

Figure A5 contains the p-values of the t-test for $\beta_{j,t} = 0$ in Nelson and Siegel (1987) model, for $j = 1, 2, 3$. Such coefficients are almost always significant (p-value < 0.05). The graphs complement the results of Figure 6 and Table 2.

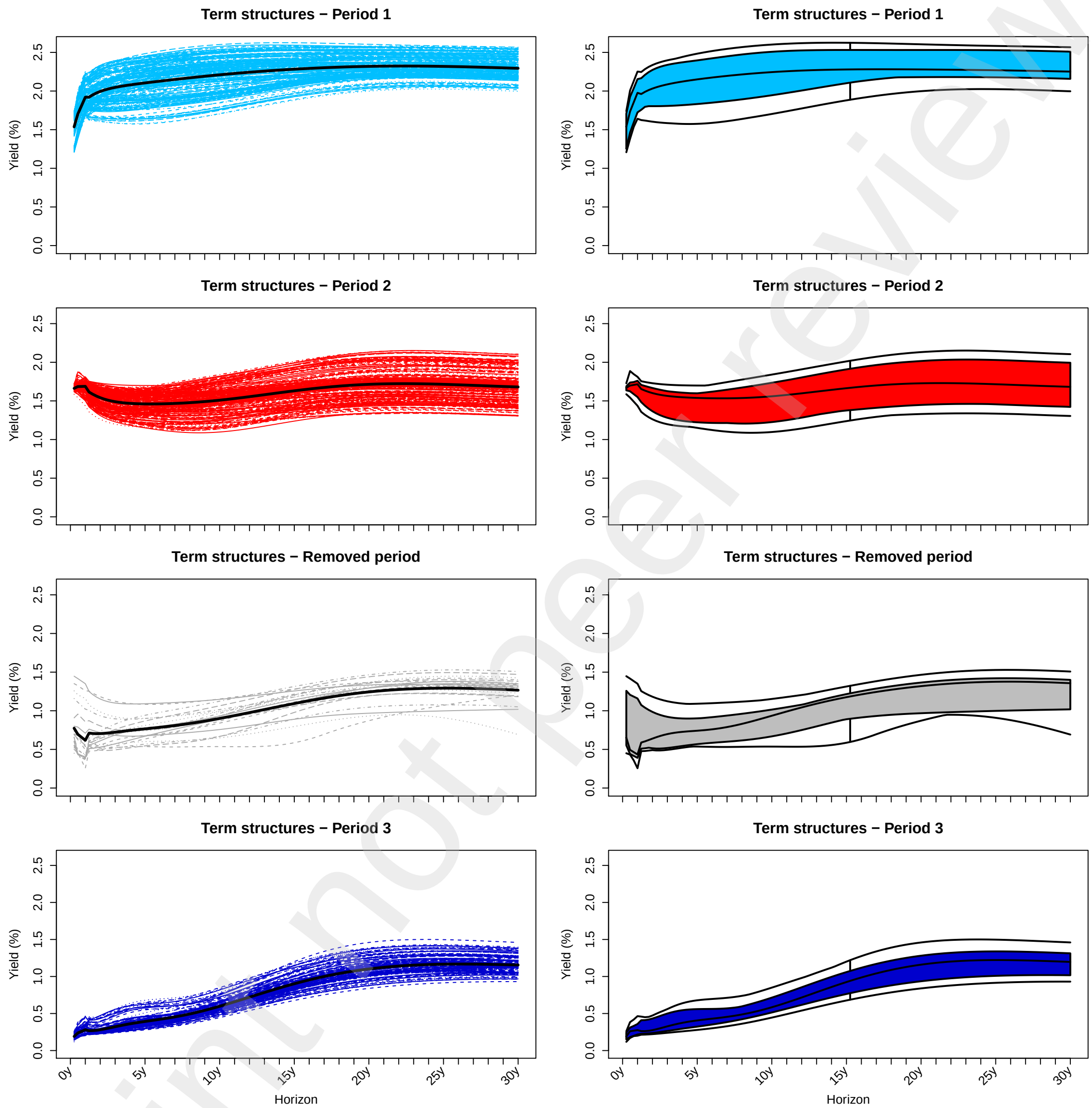

Figure A1: **Functional boxplots of the yield curves.** Daily yield curves (left panels) and corresponding functional boxplots (right panels) in the three periods considered in the analysis, as well as in the removed transition period (February 28 to March 26, 2020). The black lines in the left panels represent the mean yield curve in each period.

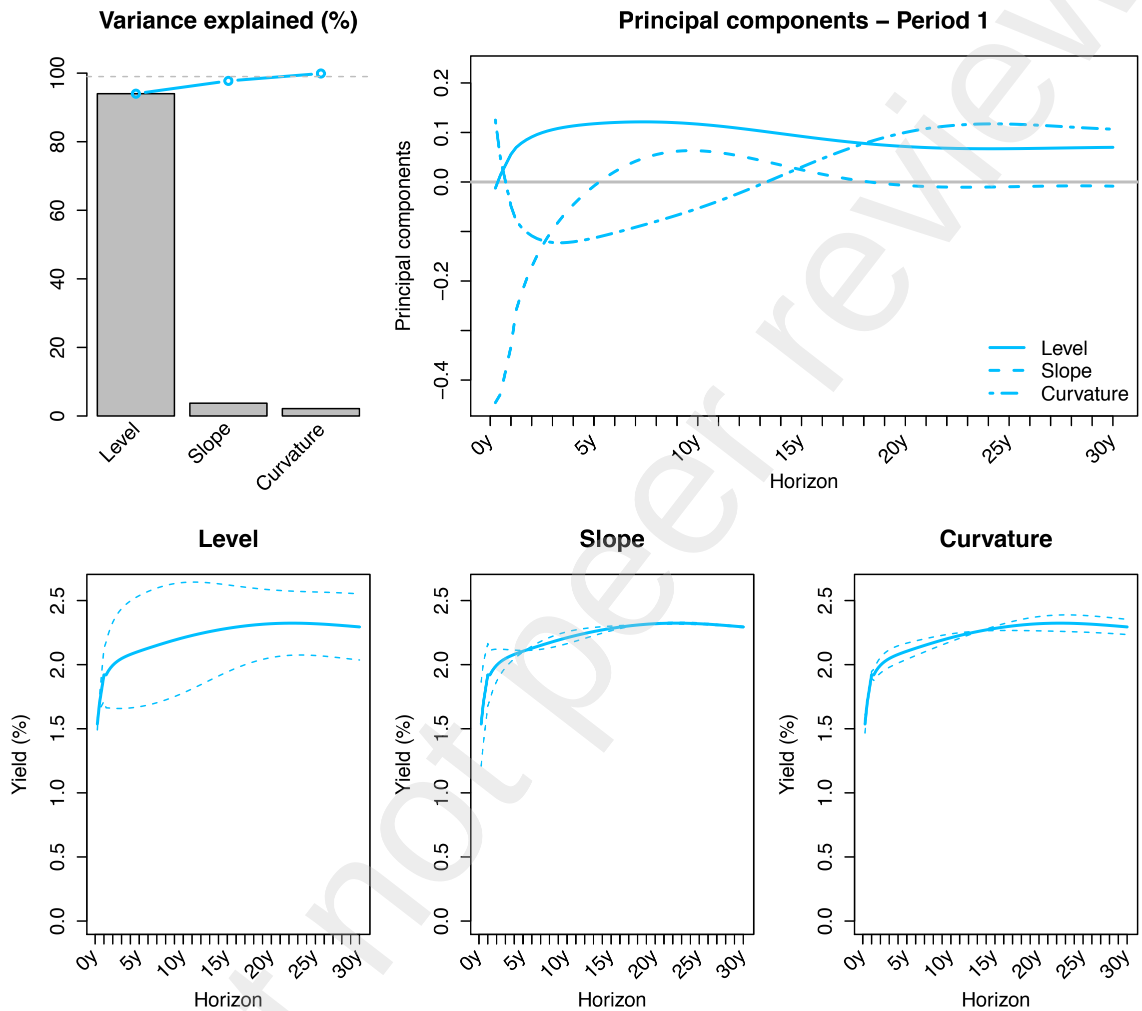

Figure A2: **(Classic) PCA results for the first period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the first period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

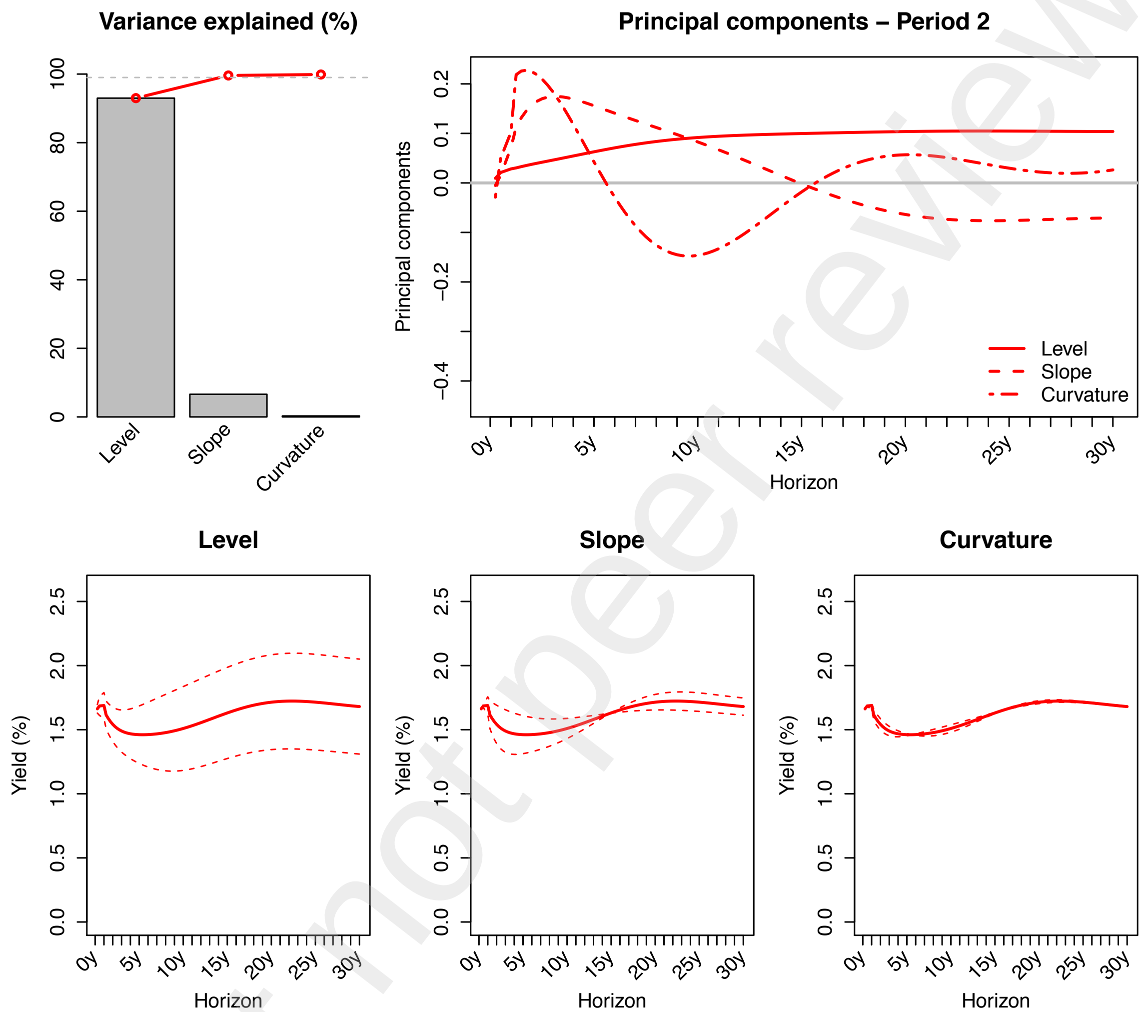

Figure A3: **(Classic) PCA results for the second period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the second period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

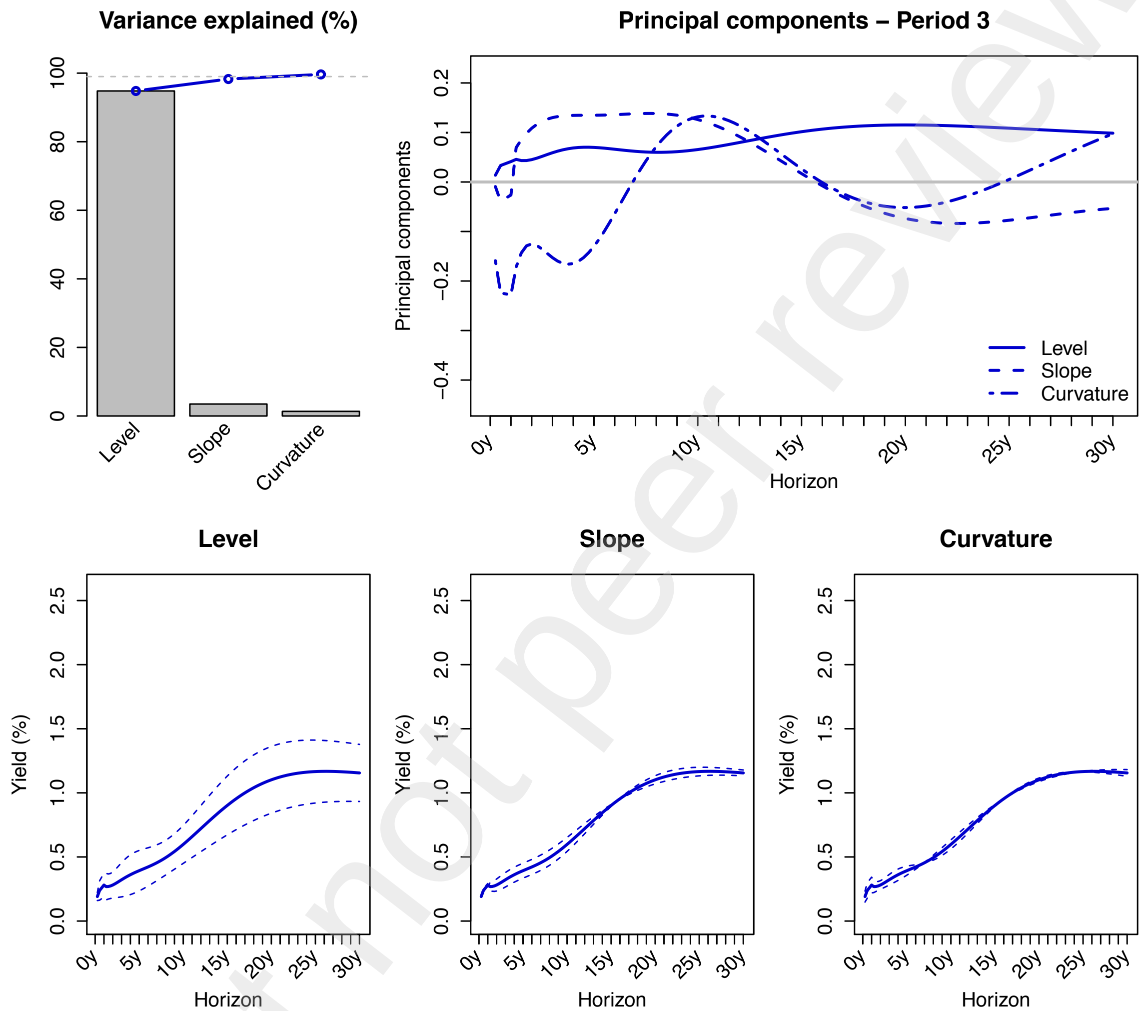

Figure A4: **(Classic) PCA results for the third period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the third period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

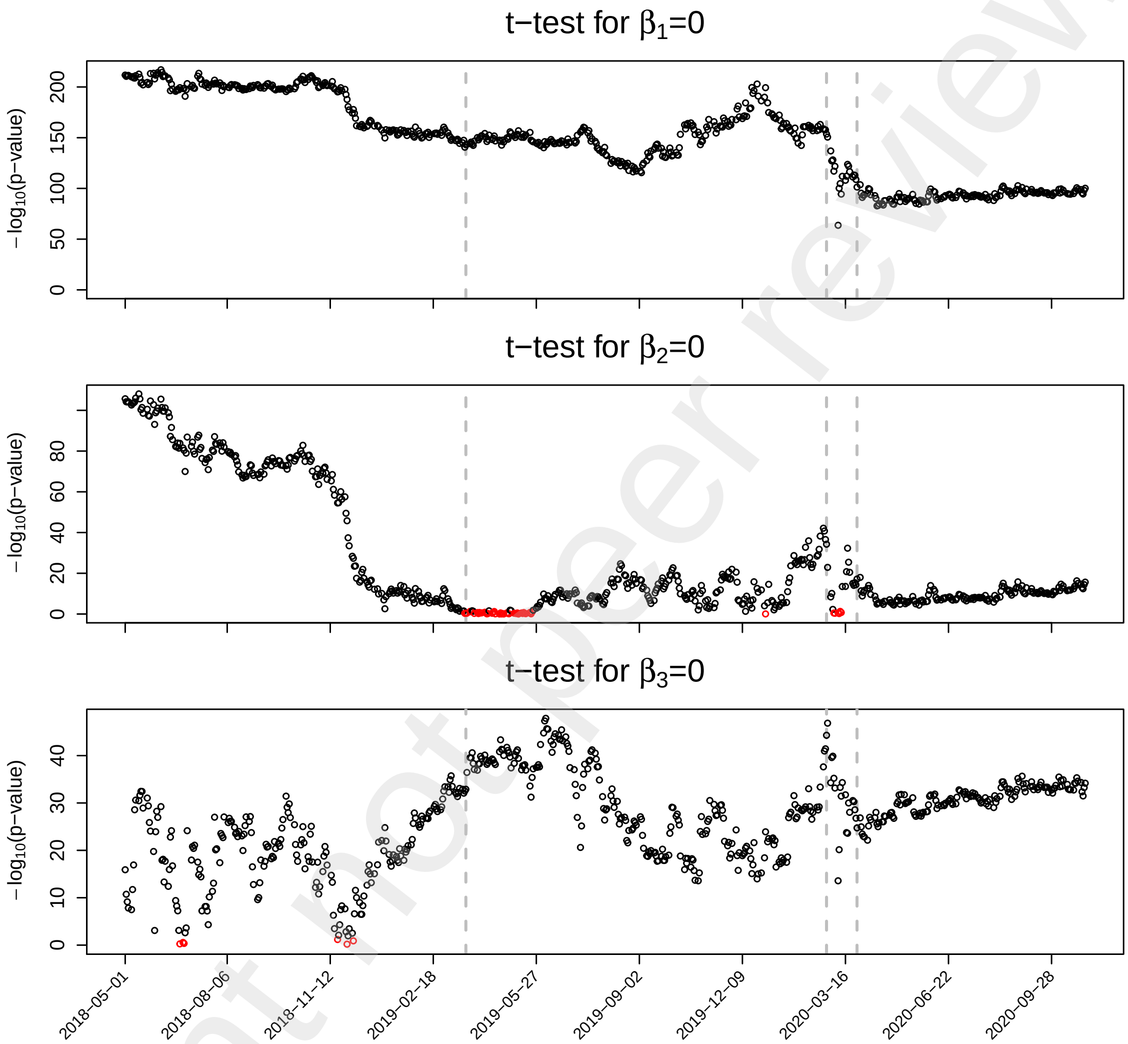

Figure A5: **P-values in Nelson and Siegel (1987) model.** P-values of t-tests for $\beta_{j,t} = 0,\ j = 1, 2, 3$ in Nelson and Siegel (1987) model. Red points correspond to non-significant coefficients (p-value > 0.05). The vertical gray dashed lines show our subdivision in periods.