Financial Regulation Reforms and Bank Capital Structure

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Abstract

I study the relationship between three regulatory measures imposed in Europe after the 2008 crisis and the capital structure decisions of regulated banks. I analyse the increase of the level of coverage of Deposit Insurance Schemes, the introduction of the Private Sector Acquisition resolution tool, and the introduction of the bail-in tool. For that, I develop a continuous time model with asset uncertainty where a bank selects the optimal capital structure while facing a regulator with a real option to impose a resolution measure. This model relies upon a set of parameters that can be estimated with empirical data and can easily accommodate additional extensions. My results show that the introduction of these regulatory measures incentivises banks to decrease the ratio of equity over assets and to increase the ratio of deposits over total debt. Lastly, introducing the bail-in tool generates a recapitalisation option, that is, banks unable to rebalance their capital structure may have an incentive to increase ex-ante the weight of bail-inable debt over total debt if they expect that a potential bail-in will result in a value-enhancing recapitalisation.

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1 Introduction

The financial crisis of 2007–08 exposed a number of weaknesses in the regulatory framework[1]. Financial institutions were revealed to be poorly capitalised and several banks experienced entirely unanticipated liquidity crises. Moreover, the legislation in place to manage bank resolution and to wind up failing banks proved completely inadequate in many jurisdictions.

New regulations were introduced around the world in response to these problems. Basel III strengthened capital requirements and introduced a countercylical buffer, deposit insurance schemes were reinforced, and a number of new approaches to bank resolution were developed. Figure 1 illustrates the effect of these regulatory changes on the capital structure of financial institutions supervised by the European Central Bank. Two major changes occurred after 2009-10. First, the ratio of equity over assets started to increase. Second, the ratio of non-deposit liabilities over total liabilities decreased: that is, deposits became an increasingly important element part of the liability structure of financial institutions.

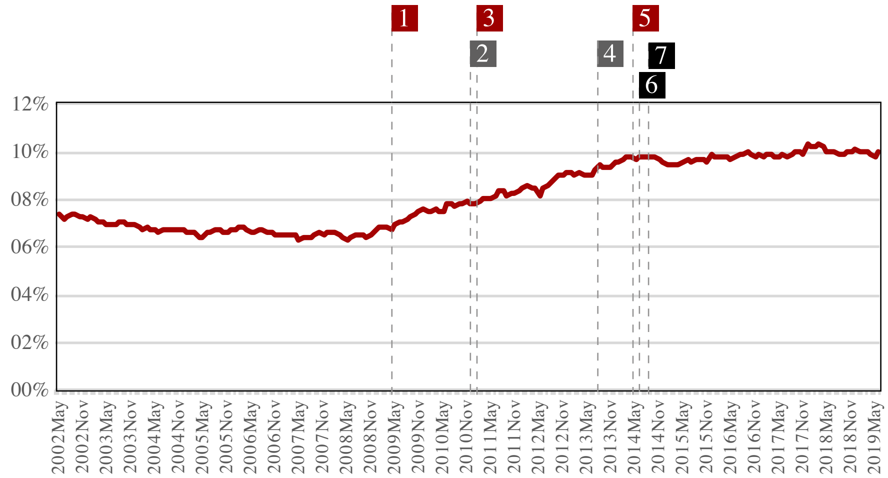
In light of these observations, this paper asks what the relationship is between the new regulatory measures and the capital structure decisions of regulated financial institutions. Specifically, I examine the effect of a significant increase in the level of coverage provided by deposit insurance schemes; the introduction of a Private Sector Acquisition resolution tool, which reduces the costs of bank resolution by enabling the transfer of assets from troubled banks without shareholder approval; and the introduction of a Bail-in resolution tool that allows regulators to convert bail-inable debt to equity without any debt-holder vote. My analysis generates predictions that are consistent with the facts presented in Figure 1. The regulatory tools that I study are detailed in Appendix A, where I present three case studies of resolution intervention.

A natural framework for considering the effects of regulatory changes on capital structure is the standard trade-off theory (Kraus and Litzenberger 1973). This theory argues that firms should trade off the tax advantages of debt against the potential costs of financial fragility. The trade-off theory captures some of the most salient determinants of a firm's in capital structure choice, and several important papers have argued that it should form the basis of my analysis of capital structure regulation (Admati et al. 2013). I deploy the trade-off theory, but I make some necessary modifications to the basic theory to reflect the fact that banks have access to insured deposits, which renders this form of debt more attractive. Banks are further complicated because their failure generates massive systemic costs that are absent from standard models of corporate finance; those costs are not considered by bank managers, and they justify some capital regulation.

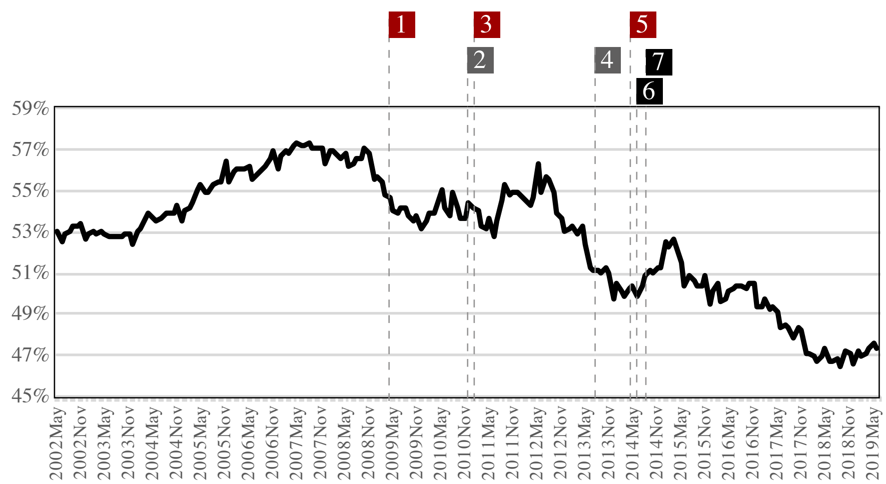
This paper makes several contributions. First, by studying how banks select their capital structure today while anticipating the future impact of potential policy interventions, it addresses the ex-ante impact of new regulations. This is particularly relevant because the regulatory changes I analyse only affect financial institutions in case regulatory intervention occurs. In contrast, recent research focusses more on ex-post implications. For example, Walther and White (2017) focus on how the set of resolution policies changes with uncertain news and possible bank-runs[2].

[1] See Dewatripont (2014) for an overview of the main issues in the banking sector and the changes in the regulatory framework that occurred after the European crisis.

[2] Walther and White (2017) use a signalling game between a regulator and uninformed depositors to study the set of bail-in policies available when facing uncertain news and potential subsequent bank-runs. Bad information constrains effective resolution as the regulator chooses weaker bail-ins because it fears a potential bank run.



(a) Ratio of Capital and Reserves over Assets of Eurozone Banks



(b) Ratio of Non-Deposit Liabilities over Total Liabilities of Eurozone Banks

Figure 1: *Subfigure (a) shows that Eurozone banks are highly levered with a ratio of equity over assets stable between 6% and 10%. This is consistent with Gropp and Heider (2010) and far below the capital ratio of non-financial firms (Frank and Goyal 2008). Bank capital capital ratios remain above the regulatory minimum, a phenomenon already documented by Flannery and Rangan (2008).*

Subfigure (b) shows a decrease in the ratio of non-deposit liabilities to total liabilities starting in 2012. This is in contrast to prior work that documents a bank preference for non-deposit liabilities (Gropp and Heider 2010). From 2002 until the end of 2012, this ratio stayed above 53%.Deposits became the dominant form of debt finance from the beginning of 2016. Banks now have more than 53 Euros of deposits for every 100 Euros of total liabilities. The pattern is approximately the same if the denominator is restricted to issued debt securities.

Data: *Balance sheet data for the total sample of banks subordinated to the European Central Bank, from May-2002 to May 2019. Source: ECB Statistical Data Warehouse.*

Second, my results shed new light on the effect that regulation has upon bank capital structure. Gropp and Heider (2010) provide empirical evidence that, in the past, financial and non-financial firms faced the same incentives when defining their capital structure. My analysis suggests that, as a result of recent regulatory changes, this is no longer the case. This paper therefore identifies new directions for empirical work on the capital structure of banks.

Third, my model relies upon a set of parameters that can be estimated with empirical data; the model therefore yields estimates of the relative magnitude and importance of a variety of economic effects. Moreover, the setup is simple and flexible and it is easy to add additional features to the base case studied below. The model should therefore be useful for regulators, think tanks, and other policy institutes.

Most of the prior theoretical work on bank capital structure analyses discrete-time, finite-horizon models. This approach has generated many important insights. However, it cannot address questions that relate to optimal closure times; nor can it generate meaningful calibrations. These lacuna can be addressed only in a continuous time framework. The continuous time framework employed in this paper is closely related to Leland (1994) and Sarkar (2013)[3] and it draws heavily upon the real options literature (Dixit and Pindyck 1994).

I present a model in which a financial institution earns stochastic cash-flows and must select a capital structure to maximise its total value. The financial institution has access to deposit finance, which is subject to the usual Diamond and Dybvig (1983)-style runs, and also to non-deposit debt finance, upon which runs are impossible. I consider a regulator that imposes a formal Basel-style insolvency regulation; it also provides deposit insurance protection, controls the cost of resolution by allowing a private sector acquisitions tool, enables bail-inable debt, and decides when to trigger the conversion of that bail-inable debt to equity.

The continuous time framework that I employ enables me to determine the optimal intervention time and to analyse in detail the ways in which banks trade off debt against equity finance, and deposit against uninsured debt finance. I am also able to perform meaningful calibration in my set-up and, hence, to generate clear regulatory guidance.

I start by considering a base case in which deposits are uninsured, resolution is privately managed, and there is a standard tax advantage to debt. In this case, the model has a single stage in which the bank operates until it is forced to close by the regulator. I solve for the bank's optimal choice of deposit and non-deposit debt financing and I derive a closed-form solution by trading off the costs of bankruptcy against the tax benefits of debt finance. I impose a no-run constraint upon the capital structure: the bank is required to hold a minimum level of non-deposit finance to cushion depositors from bankruptcy and disincentive bank runs; beyond that level, banks are indifferent between deposit and non-deposit debt finance.

I introduce additional elements to the base case model in stages. First, I consider the effect that costless deposit insurance has upon the optimal capital structure. Second, I examine a private sector acquisitions tool that causes a discrete decrease in the costs of bankruptcy. Finally, I add a bail-in tool that allows the regulator to impose a debt-to-equity swap.

The bail-in tool introduces a new stage into my model. The first stage runs from the bank's choice of capital structure to the time the regulator forces the conversion of unprotected creditor claims into equity. Conversion occurs at a rate that respects the "no worse off" condition required

[3] As in Leland (1994) and Sarkar (2013), my capital structure depends on the trade-off between debt tax shields and bankruptcy costs. However, I depart from these models by accounting for different types of debt, by having an endogenous intervention trigger determined by the regulator and, more importantly, by considering that the bank faces the aforementioned resolution policies in case of intervention.

by current resolution frameworks[4]. The second stage runs from the conversion until the time when the regulator closes the firm. Closure occurs using the private sector acquisition tool combined, if necessary, with deposit insurance coverage. I provide closed form solutions for each type of debt.

My analysis yields a number of empirical implications. First, the model predicts that each of the regulatory changes that I consider causes an increase in the leverage of financial institutions. An increase in deposit insurance protection renders debt cheaper and so should cause an increase in leverage; a private sector acquisition tool reduces the cost of closure and, hence, also pushes banks towards debt financing; and finally, a bail-in postpones or entirely avoids formal insolvency or resolution procedures and, hence, by reducing the costs of financial fragility, also serves to increase bank indebtedness.

Second, calibrating the model suggests that the observed increase in banks' equity/assets ratios since 2010 (Figure 1.a) can only be explained by the increase in regulatory capital requirements introduced by Basel III[5]. Moreover, the sensitivity of bank's equity/assets ratio to increases in capital requirement ratios may be higher than apparent. Introducing these resolution tools and increasing the limits of deposit insurance push the institution towards debt financing, which undermines the effect of a capital requirement increase.

Third, the model also generates predictions for the mix of deposit and non-deposit debt finance that banks use. The predictions are consistent with the debt structure trend for European banks presented in Figure 1.b: between 2012 and 2018, the long-standing bank preference for non-deposit liabilities over deposits was reversed. In line with this fact, my model predicts that recent regulatory changes should leave banks with more deposits than non-deposit liabilities, because deposit insurance relaxes the no-run constraint and subsidises depositors in the event that the bank closes.

Interestingly, my analysis demonstrates that deposit insurance need not cause deposits completely to crowd out non-deposit liabilities. It is hard for any firm to change its capital structure in a near-death situation (Gertner and Scharfstein 1991). The bank therefore takes advantage of the bail-in tool by selecting an ex-ante level of bail-inable liabilities that can be later converted to equity. This policy relaxes the closure constraint for lower firm performances, and so gives the bank's assets time to recover. That is, bail-ins generate a *recapitalisation option*. The recapitalisation option is valuable because the bank selects a fixed capital structure up-front. More generally, the recapitalisation option is valuable whenever the bank's capital structure is sticky; this could occur either because of adjustment costs or because of regulatory strictures. My calibration results indicate that banks with high growth rates of earnings, low volatility of earnings, low taxation and facing high capital requirements always hold non-deposit liabilities. Moreover, my model predicts that, conditional on high growth rates of earnings, financial institutions with high uncertainty will choose very high levels of non-deposit liabilities.

My analysis suggests that policy makers wanting to introduce the bail-in tool with a deposit insurance scheme should also impose a minimum volume of bail-inable liabilities[6]. My calibration shows that different growth rates of cash-flows, volatility, taxation and regulatory requirements may

[4] The No Worse-Off Principle is defined in Article 74 of Directive 2014/59/EU. See Bank Recovery and Resolution Directive for details.

[5] The underlying ideas of Basel III were initially discussed in 2009. The European Union adopted a legislative package in 2013 increasing some capital requirement ratios and introducing new types of capital requirements. See Basel III Summary Table.

[6] The BRRD indicates that the resolution policies are conditional on the minimum requirement for own funds and eligible liabilities (MREL). In detail, banks in the European Union for which liquidation is not the preferred strategy may be required to hold a minimum level of bail-inable debt to be suitable candidates for bail-ins. MREL requirements are still under development at the time of this paper. The 2008 guidelines for the first wave of resolution plans can be found here.

entail different optimal debt structures for financial institutions. I find that it is not guaranteed that all financial institutions will choose to hold bail-inable debt at the levels deemed sufficient by the regulator to make this tool useful. Some banks simply do not hold bail-inable debt and, hence, have no recapitalisation option.

Moreover, imposing a minimum volume of bail-inable liabilities attenuates the solvency and liquidity risks that high levels of deposits may pose to Deposit Insurance Funds. My model does not incorporate systemic risk but I hypothesise that, if a majority of banks exhibit the preference for deposits that the banks in my model do, then the deposit insurance fund may be threatened by negative shocks to the banking sector. In this case, resolution tools aimed at sterilising bailout needs are innocuous because the sovereign will respond to them by bailing out the deposit insurance fund instead of the financial institutions.

The roadmap for this paper is as follows. Section II presents a review of related literature; Section III introduces the base case, that is, before the regulatory changes; Section IV introduces costless deposit insurance to the base model; Section V adds private sector acquisitions; Section VI adds the bail-in tool; Section VII provides quantitative results and comparative statics; and Section VIII concludes the paper and suggest future research avenues.

2 Related Literature

▷ **Trade-Off Theory of Capital Structure.** My model builds on the trade-off theory of capital structure, first studied by Kraus and Litzenberger (1973). This theory identifies the optimal corporate capital structure as arising from a trade-off between the tax shield of debt and deadweight costs[7]. See Frank and Goyal (2008) for an overview of trade-off theory, Sundaresan (2013) for a review on capital structure and a summary of stylised empirical facts, and Graham et al. (2015) for an empirical overview of corporate financing policy over the last century.

The importance of tax benefits of debt for the capital structure decisions of financial institutions was often neglected in the prior banking literature. That changed recently with several empirical works confirming the importance of tax shields for the capital structure of banks. Schandlbauer (2017) adjusted the setup of Heider and Ljungqvist (2015) for US financial institutions subjected to 13 different state tax increases between 2000 and 2011. They find that well-capitalised banks increase their non-depository debt to benefit from an enlarged tax shield of debt. Financial constraints over worse-capitalised banks prevent these institutions from making the same size and type of adjustment. Instead, they partially adjust their hybrid securities to benefit from tax deductions. Furthermore, Schepens (2016) studies an exogenous decrease in the relative tax benefit of debt to equity for Belgium banks in 2006. This effective increase in the tax shield of equity increased the capital structure (equity over total value). In my model this would correspond to to a negative shock in the tax rate parameter. Lower marginal benefit of debt leads to a higher capital structure ratio. For additional evidence on the importance of taxation to banks' capital and debt structure decisions see Ashcraft (2008), Horvarth (2013), Hemmelgarn and Teichmann (2014), de Mooij and Gaetan (2015), and de Mooij and Keen (2016). For general reviews on the impact of taxation in non-financial and financial firms see Hanlon and Heitzman (2010) and Graham and Leary (2011).

The second important pillar of the trade-off theory is the magnitude of deadweight costs. Empirical evidence for financial institutions is not extensive but it is well established since the Savings and Loan Crisis. For instance, James (1991) studied the realised losses of bank failures from 1985 and 1988 using FDIC data. They find that bankruptcy costs amount to approximately 30% of failed bank's assets. Furthermore, whole bank transactions, where total assets are purchased by another institution, yield the lowest average loss among the different types of resolution tools available to the FDIC. Evidence on non-financial firms is particularly abundant. Andrade and Kaplan (1998) studied a sample of financially distressed highly leveraged transactions. Using market data, they estimated distress costs of around 10 to 20% of firm value, with a maximum cap at 23%. Korteweg (2010) estimates 15% to 30% of firm value for a sample of highly distressed LBO firms. Davydenko et al. (2012) proposes a novel approach that is not limited to highly leveraged transactions. They account for shocks in equity and debt value caused by unanticipated information in default announcements. They estimate an average default cost of around 21.7% of the market value of assets, with costs of 30.5% in the case of bankruptcies. More recently, Glover (2016) used a structural model with state dependency and calculates default cost ex-ante and ex-post bankruptcy. For a sample of 2505 firms, the average company faces an expected default loss of 45% of firm value. However, conditional on default the average cost amounts to only 25%, in line with previous research. For additional studies on the problematic of financial and economic distress and corporate bankruptcy see, for example, Asquith et al. (1994), Bris et al. (2006), Acharya et al. (2007) and

[7] This theory originally rose from the discussion about the extreme predictions of Modigliani and Miller (1963), namely that, a firm with a linear value function will finance with 100% debt when subject to corporate income taxation.

Almeida and Philippon (2007). For related surveys, see Hotchkiss et al. (2008) and Senbet and Wang (2012).

From a modelling perspective, capital structure literature is divided in two groups: Static and dynamic. The static trade-off theory supposes that the firm's debt level is optimised at a single period. The model of Bradley et al. (1984) provides the standard setting for the static capital structure under opposing effects of taxation and bankruptcy costs. Under a dynamic approach firms adjust the capital structure dynamically throughout time. The inception of dynamic models occurred with Brennan and Schwartz (1984) who provided a dynamic trade-off theory framework under stochastic shocks. The absence of adjustment costs generates a dynamic adjustment to the optimal levels every time the firm suffers a shock. However, this adjustment should not occur at every point in time in order to be realistic. Hence, Fischer et al. (1989) introduced adjustment costs. In this case, leverage ratios are only adjusted when the benefit from rebalancing the capital structure exceeds adjustment costs. They find even when these costs are small, the firm's debt ratio can have significant changes over time. More recently, Strebulaev (2007) builds and calibrates a dynamic capital structure model with infrequent adjustment to generate data and compare with existing empirical literature. Standard cross-sectional results seem to corroborate a number of empirical studies and the model is rejected using established methodology to test results. Overall, the paper reinforces the need to expand the research on capital structure. Examples of other developments in capital structure models includes endogenous investment (Hennessy and Whited 2005), a principal-agent setting with agency costs (Demarzo and Sannikov 2006), effects of shifts in macroeconomic states of the economy (Hackbarth et al. 2006).

▷ **Distress and Private-side Restructuring Literature.** The sequence of Bail-In and Private Sector Acquisition in this model is closely related to private-side[8] restructuring literature. Restructuring decisions are usually endogenous and depends on a bargaining distressed exchange of claims between stakeholders. I do not account for bargaining between players. I respect the BRRD regulation and consider that unprotected creditors do not have an ex-ante value loss associated to a future Bail-in conversion as the regulator is independent from stakeholders' lobbying.

However, private-sided restructuring measures can erode credit claims' value. For example, Mella-Barral and Perraudin (1997) shows that strategic debt service affects corporate debt valuation. In this model, shareholders have negotiation power. They confront creditors with a "take-it-or-leave-it" coupon payment when the firm is in financial distress. Creditors unable to confront this suffer a value transfer to shareholders. In Mella-Barral (1999) shareholders and creditors face a distressed renegotiation in the form of a debt swap. High-leveraged firms can experience credit reorganisation before liquidation, while low leverage firms are directly liquidated. Creditor's bargaining power affects the price of their claims. Lambrecht (2001) extends the debt swap problem to a duopolistic framework. Consistent with Zingales (1998), they show that the last firm to go bankrupt has higher relative operating profits combined with lower debt repayments, and higher expected value of becoming a monopolist, i.e. the "survivor".

Sundaresan and Wang (2007) considers a Nash bargaining negotiation between shareholders and creditors while the firm is an ongoing concern. Creditors with high bargaining power can capture some of the firm value before reaching bankruptcy. Coupon reduction depends on fire sales costs, bargaining power and taxes. In the model of Sarkar (2013), shareholders make a distressed exchange offer to creditors near financial distress in order to postpone costly bankruptcy reorganisation. Parties negotiate via a Nash bargaining game where the bargaining power is split between shareholders

[8] The voluntary measures used by the firm's stakeholders to deal with bankruptcy.

and creditors, thus allowing for the possibility of renegotiation to split the restructuring surplus. They show that greater bargaining power resulting in an early DE offer, smaller debt reduction and smaller equity stake. See Hotchkiss et al. (2008) for an empirical research survey on the use of private measures for resolving default and reorganising financially distressed firms.

▹ **New Regulation and Contractual Arrangements.** My article fits within a larger literature that investigates the impact of recent regulatory changes on the decisions of the bank's stakeholders. Post-financial crisis literature on capital structure paid initial attention to the use of contingent capital, namely contingent convertible (CoCos) debt. The conversion threshold is a central concern in these models as this parameter will dictate the conversion of CoCo debt to equity. For example, Glasserman and Nouri (2012) study contingent capital in the form of debt subject to equity conversion upon reaching a threshold based on book value variables. Their results indicate that the fair yield for contingent capital is sensible to the convertible tranche, asset's volatility and recovery rate. Chen et al. (2017) follows with a model with endogenous default, debt rollover and a combination of jump and diffusion processes. They conclude that bankruptcy induced by excessive debt, which occurs when CoCos are combined with endogenous default for low threshold conversion, can be overcome if parties agree to hold sufficiently high debt-to-equity conversion thresholds. My intervention triggers are close in spirit to the conversion thresholds presented in these two papers. However, I derive it optimally from the regulator's free-boundary problem in which the regulator has to choose the best time to step in the bank. As a result, uncertainty plays a key role in every intervention trigger which induces the regulator to delay intervention if the bank's assets are more volatile. Chen et al. (2017) has a similar mechanism for one of the thresholds, i.e. the bankruptcy threshold chosen by the shareholders. However, I differ in the way I treat the order of triggers. In my model, default is impossible to occur before conversion when the firm has junior debt in its debt structure. That is because the regulator is in charge of the stopping times and will impose a bail-in in that case. This may not occur in a model with contractually agreed contingent convertible's thresholds. Differently, Sundaresan and Wang (2015) focused on CoCos with market equity triggers in which shareholders and junior creditors are do not hold an option to select their optimal conversion policies. As a result, this restriction of interests lead to the existence of multiple equilibrium. For a unique equilibrium to hold, a conversion rate must not lead to a value transfer from shareholders to junior creditors. However, that prevents punishing shareholders for firm mismanagement and leads to problems when designing the CoCo contract with market triggers. Also, the issue of equilibrium multiplicity worsens in the presence of discrete jumps and increasingly costly bankruptcy. Additional discussions of trigger designs can be found in Baily et al. (2009) for a combination of a systemic crisis indicator and bank-specific accounting information, in Glasserman and Wang (2011) and Bolton and Samama (2012) for bank management's decision, and in Dickson (2010) for regulatory decision.

Furthermore, post-financial crisis literature on capital structure focused more recently on the new regulatory environment of financial institutions. For example, my paper is closely related with Sundaresan and Wang (2017). They study the case of liability structure in a classic capital structure model with deposits and non-deposit liabilities. Optimal funding decision builds on the trade-off between debt tax shield and bankruptcy costs. This shares some features with my paper though I opt to extend to the impact of resolution tools, effects of the bail-in tool, instead just the introduction of deposits in the capital structure. We both find that, in equilibrium, financial institutions want to hold more deposits in their capital structure once deposit insurance is introduced. Lambrecht and Tse (2018) present a continuous time model where a risk-averse manager dynamically optimises the capital structure and the asset's riskiness. They study how liquidation, bailout and bail-in policies affect these decisions. My model is distinct in two aspects: First, I focus on a static capital structure optimally determined at time $t = 0$. A static capital structure is, I believe, more adequate since firms do not actively correct their capital structure when facing shocks in their market value (Welch 2004) and, in a near bankruptcy situation capital structure changes are difficult to implement (Gertner and Scharfstein 1991). Also, the model becomes tractable with a static capital structure, allowing me to solve for the intervention triggers and to obtain closed form solutions for the equilibrium controls. Second, I optimise for total bank value instead of the managerial value (which depends on the equity value). This impacts optimal decisions if the bank faces liquidation in bad states of the nature. In Lambrecht and Tse (2018) model, the decision maker increases exposure by issuing bad loans. In my case, the bank decreases leverage because optimisation accounts for the creditors loss in default. I also relate with the work of Hugonnier and Morellec (2017), They study the impact of liquidity and leverage requirements on default conditions of financial institutions. They show that liquidity requirements decrease the size of losses conditioned to default but increase the probability of default. They show also that leverage requirements decrease the probability of default. Therefore, a combination of both requirements, as required by Basel III, can simultaneous decrease losses in default and the likelihood of default.

▷ **Bank Runs and Deposit Insurance.** My framework imposes a minimum non-deposit liabilities to prevent bank-runs in the absence of a deposit insurance scheme. This is supported on the extensive literature of depositors runs. For instance, the seminal paper Diamond and Dybvig (1983) studies how different types of depositors act in the presence of demand-deposit contracts. In the welfare improving equilibrium, impatient depositors withdraw earlier at the cost of patient depositors. In the bad equilibrium, a bank run occurs when both type of investors try to simultaneously withdraw their holdings from the bank. Nonetheless, this work leaves unanswered the question of which factors trigger the run. Information about the lack of quality of the bank is not necessarily the trigger as shown by Breeden (1984). In this model, highly averse depositors may choose to consume less now to invest more in an uncertain future, regardless of bank asset's quality. Intuitively, Jacklin and Bhattacharya (1988) shows that bank runs do not threaten the bank if long-term assets are liquid and depositors are not significantly risk averse. Goldstein and Pauzner (2005) extend Diamond and Dybvig's work by exploring the impact of uncertain economic conditions on equilibria. The bank equilibrium contract emerges as a function of the bank-run probability. Allen et al. (2018) follows by considering deposit insurance as a public policy. They find that government guarantees mitigate the inefficiencies created from lack of coordination between the types of depositors. They note that not all types of guarantee schemes are equally effective. Guarantees against panic runs are the most effective in the mitigation of bank runs at no public expense. Political environment tend to affect the adoption and design of government deposit guarantee systems (Demirguc-Kunt et al. 2008). More democratic countries with fewer underlying risks increase government deposit insurance. This seems to correlate positively with the influence of international institutions.Deposit insurance can, however, trigger moral hazard frictions as demonstrated by Chernykh and Cole (2011) in an experiment in Russia[9]. Demirguc-Kunt and Detragiache (2002) argues that effective prudential regulation and supervision mitigates moral hazard frictions and other bank fragilities exacerbated by deposit insurance. Furthermore, Morrison and White (2011)'s model of moral hazard and adverse selection study the ideal deposit insurance. Contrary to the idea that banks should pay for deposit

[9]They concluded that banks entering the scheme increased risk-taking, sustaining the theory that deposit insurance can motivate moral hazard. Also, banks observed an increment on their level of deposits in absolute terms and relative to total assets, which benefitted smaller banks. State-owned banks presented a decrease in deposits, suggesting that the shift in risk did not affect depositors.

insurance, they show that general taxation combined with tough capital requirements is the best configuration. See Allen et al. (2011) and Calomiris and Jaremski (2016) for additional reviews on deposit insurance.

3 Base Case

3.1 Elements of the Model

I consider a model with two active players: a Bank and a Regulator. The players move sequentially: The Bank selects its capital structure, and then the Regulator chooses when to close the bank. My base case is close to the standard Leland (1994) framework.

▷ **The Bank.** The bank receives cash-flows[10] [11] represented by an Ito diffusion $x = \{x_t : 0 \leq t < \infty\}$ that satisfies the following Geometric Brownian Motion:

$$dx_t = \mu x_t dt + \sigma x_t dw_t, \tag{1}$$

where $x_0 > 0$, $\{w_t\}_{t \geq 0}$ is a Wiener process, μ is the instantaneous conditional expected growth rate of x and $\sigma > 0$ is the instantaneous conditional standard deviation. I write $\rho > 0$ for the discount rate and, for convergence, I assume that $\rho > \mu$.

The bank's assets are financed with liabilities and equity. The liability part of the bank's balance sheet comprises two types of perpetual debt: non-deposit liabilities and deposits. The bank decides its liability structure at time $t = 0$ by selecting coupons c for non-deposit creditors and d for deposit creditors. I write $l = c + d$ for the total coupon paid by the bank.

Assumption 1 (Non-Negative Coupons)**.** *The bank selects non-negative coupons for non-deposit liabilities and deposits.*

In my model, debt is used to finance assets and, therefore, the bank is expected to pay a positive interest payment every time it receives cash-flows.

The value $V(x_t)$ of the bank's assets is the present value of future cash-flows:

$$V(x_t) = \mathbb{E} \int_t^{+\infty} e^{-\rho(s-t)} x_s ds = \frac{x_t}{\rho - \mu} \tag{2}$$

and the accounting value L of the bank's liabilities is the present value of future coupon payments:

$$L = \int_t^{+\infty} e^{-\rho(s-t)} (c + d) ds = \frac{c + d}{\rho} \tag{3}$$

The regulator may force the bank to close at any time, in which case I say that the bank has entered bankruptcy. If that happens, the bank's assets are sold to another owner, who continues to receive the cash flow stream x_t. If the bank goes bankrupt, then it experiences a cost equal to a fraction $\epsilon \in (0, 1)$ of the value $V(x_t)$ of its assets at the time of its bankruptcy: that is, it experiences a

[10] I select a single asset case for simplicity. However, Petersen and Mukuddem-Petersen (2005) and Witbooi et al. (2011) provide models with composite of risky assets.

[11] Moreover, there can be an issue of having unobservable firm's asset value. In this case, to determine intervention triggers using real world data can be problematic if a firm has unobservable asset's cash-flows. That can be overcome by using credit spreads or the CDS risk premium as seen, for instance, in Yang and Zhao (2015) and Pelger (2012).

cost $\epsilon V(x_t)$. I refer to the value of bank assets net of the costs of bankruptcy as the *impaired asset value*[12], $(1-\epsilon)V(x_t)$.

The bank also pays corporate tax at a rate[13] $\tau \in (0,1)$ on cash-flows above its debt payments. It follows that equity-holders receive a continuous income stream after interest and tax of $(x_t - c + d)(1-\tau)$.

The bank selects c and d at time $t = 0$ in order to maximise the value function $B(x_t)$. $B(x_t)$ is the sum of the values of the claims on the bank:

$$B(x_t) = E(x_t) + C(x_t) + D_t$$

where $E(x_t)$ is the value of the claim held by the equity-holders (or shareholders), $C(x_t)$ is the value function for non-deposit creditors and D is the value function[14] of depositors. These value functions are specified in Subsection (3.3). I write $CS(x_t)$ for the capital structure ratio: that is for the ratio between equity value and bank value. I define the debt ratio $DS(x_t)$ to be the ratio between the non-deposit creditors' value function and the total value of all liabilities:

$$CS(x_t) := \frac{E(x_t)}{B(x_t)} \qquad\qquad DS(x_t) := \frac{C(x_t)}{C(x_t) + D}$$

I impose the following assumption:

Assumption 2. *All claimants have perfect information about the bank's cash-flow.*

Assumption 2 ensures that bank runs are possible in this model. Depositors worry about solvency[15] and so, if depositors observe that the present value of the bank's cash-flows is insufficient to cover their claim, they withdraw their funds. The private cost of bank runs is so high that the bank prefers to avoid it. Hence, I assume:

Assumption 3 (No Bank-Run)**.** *The bank selects a capital structure that does not induce bank-runs.*

▹ **Regulation.** The bank is subject to capital regulation. Two elements of the bank's balance sheet are important for capital regulation: the value of assets presented in condition (2) and the accounting value of liabilities in condition (3). The regulation stipulates a safe value $\xi \in (0,1)$ for the percentage by which the value $V(x_t)$ of assets exceeds the accounting value of liabilities. That is:

$$\frac{V(x_t) - L}{V(x_t)} \geq \xi \tag{4}$$

The regulator's only possible action is to close the bank. Banks are socially valuable and, ceteris paribus, regulators prefer not to close them. The regulator need not close a bank as soon as the

[12]This paper treats bankruptcy costs as fire sales costs, in a similar manner as Leland (1994) and Strebulaev (2007). Empirical literature shows that selling assets at discount is frequently used by distressed firms (Asquith et al. 1994, Pulvino 1998). More recently, Ma et al. (2019) have shown that innovative firms under distress tend to sell first intangible assets like patents before liquidating fixed assets.

[13]It is simple, though outside my goal, to extend this model in order to account also for a dividend tax rate, as in Tserlukevich (2008).

[14]The depositors' value function does not depend on x_t.

[15]I focus on solvency concerns rather than liquidity concerns which is more usual in bank-run literature. Nonetheless, these two options should not differ significantly in my setup.

capital requirement (4) is violated[16]. However, keeping a bank open when the capital regulation is not satisfied is risky. I capture this effect by assuming that the regulator experiences a flow of utility $R(x_t)$ given by the margin by which Equation (4) is satisfied:

$$R(x_t) := V(x_t)(1-\xi) - L \tag{5}$$

Hence, when capital regulation condition (4) is violated, the regulator experiences a negative utility. It can stop the negative utility flow by closing the bank. However, if the regulator chooses to leave the bank open, there is a chance that it will recover, and that the utility flow $R(x_t)$ will turn positive. The regulator therefore faces a real options problem: it must select the optimal time to close the bank so as to trade off the benefits of immediately preventing further loss of utility when condition (4) is violated against the possible benefits to be earned if condition (4) is satisfied again in the future. I write T_b for the optimal closure time. Peskir and Shiryaev (2006) demonstrate that T_b is the first passage time at which x_t crosses a boundary value x_b:

$$T_b = \inf\{t \in [0, +\infty) : x_t \notin (x_b, +\infty)\} \tag{6}$$

I write the closure option value as $O(x_t)$ and I value $O(x_t)$ in subsection 3.2. More importantly, I also derive the closed form of the regulator's closure trigger x_b, which the bank incorporates in its optimal capital structure problem.

I impose the following assumption:

Assumption 4 (Initial Capital Requirement)**.** *The bank respects regulatory condition (4) when it chooses the capital structure.*

▷ **Timeline and Equilibrium.** Consider Figure 2. At $t = 0$, the bank selects coupons $(c, d) \in [0, +\infty)^2$ so as to maximise its value function $B(x_0)$. The bank operates until the regulator chooses to close the bank[17]. I therefore consider two stages. During stage 1, $t \in [0, T_b)$ and the bank is operational. The second, *closure* stage, obtains for all $t \in [T_b, +\infty)$. I use a subscript $n \in \{1, \mathcal{C}\}$ to represent each stage. For example, $B_{\mathcal{C}}(x_t)$ is the bank value in the closure stage.

The model of this paper is a sequential game with perfect information and I therefore solve it by backward induction. I write the equilibrium tuple as $\mathcal{E}$:

$$\mathcal{E} := (c^*, d^*, x_b^*)$$

[16] I use the current European banking crisis management framework as inspiration for the actions of my regulator. In Europe, banks are subject to early requirements spanning internal governance, business strategy, capital and liquidity adequacy, as detailed in Art. 27 of 2014/59/EU and in EBA/GL/2015/03. If a bank is not fulfilling these requirements, the regulator asks the bank to implement early intervention measures. These are managerial and operational changes that can recover the bank and avoid triggering a harsh resolution measure such as closure. A resolution is only triggered when the regulator determines the bank is failing or likely to fail (Art. 32 of 2014/59/EU). Failing own fund requirements (Art. 104 of 2014/59/EU) and having less assets than liabilities are objective measures that contribute to triggering resolution but, according to EBA/GL/2015/07, such decision is "an expert judgement and should not be automatically derived from any of the objective elements". In general, the regulator decides to trigger a resolution when no early measures are expected to restore the bank to viability within a reasonable timeframe.

[17] I depart from Sundaresan and Wang (2007), Sarkar (2013) and related works by having a separate entity that makes the intervention decision instead of allowing the shareholders to define the moment of bankruptcy. In these works, conflicts of interests would lead to the typical Leland (1994) bankruptcy trigger, later than the timing that would fulfil the typical interests of a regulator. In my case, the capital requirement buffer contributes to an earlier preemption than the equivalent setup of these models.

At time 0, the bank chooses the optimal periodic coupons c^* paid to non-deposit creditors and d^* paid to depositors. The regulator closes the bank at time T_b, when the cash-flow x_t first crosses the closure trigger $x_b^* := x_b(c^*, d^*)$.

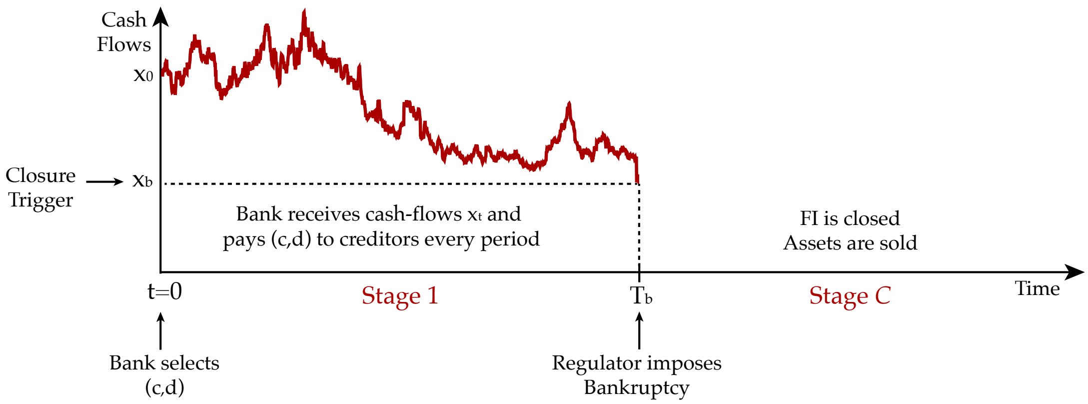


Figure 2: ***Base Case Timeline.*** *Cash-flows evolve stochastically while gradually declining until they reach the closure trigger x_b. That occurs at time T_b, when the regulator closes the bank and it enters bankruptcy.*

3.2 The Regulator's Decision

In this section, I solve for the regulator's closure option $O(x_t)$ and the optimal closure trigger x_b. The regulator closes the bank when option value $O(x_b)$ net of the disutility $R(x_b)$ is equal to the utility 0 of closing the bank; that is, when the following expression is satisfied:

$$O(x_b) + R(x_b) = 0$$

When the expression is satisfied, the regulator is indifferent between keeping the bank alive and closing it.

I determine the value $O(x_t)$ of the regulator's closure option in Appendix B, using standard real options techniques laid out by Dixit and Pindyck (1994). I find that, for a given closure trigger x_b,

$$O(x_t) = -R(x_b)\left(\frac{x_t}{x_b}\right)^{\beta}, \qquad \forall x_t \geq x_b \tag{7}$$

where

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0 \tag{8}$$

Condition (7) states that the value of the regulator's closure option is the product between the regulator's payoff if the option is exercised ($-R(x_b)$) and the expected discount factor[18] $(x_t/x_b)^{\beta}$ that discounts the payoff from time T_b to the current time.

[18] The expected discount factor $\mathbb{E}\left[e^{-\rho T_b}\right]$ is used to discount payoffs earned at a random time T_b. As shown by Dixit and Pindyck (1994), I can use the fact that T_b is the first time x_t crosses x_b to write it as $(x_t/x_b)^{\beta}$.

In this framework, β is the elasticity[19] of the closure option with respect to the cash-flow x_t: that is, β measures the relative change of the option's value to relative changes in cash-flows. β is negative so that, as one would expect, the value of the closure option decreases as cash-flows increase.

The regulator closes the bank when the closure option has the highest value. The closure trigger x_b therefore solves the following problem:

$$x_b \in \arg\max_{x_b} O(x_t),$$

The following Proposition, which I prove in Appendix C, presents the solution to this problem:

Proposition 1 (Optimal Bank Closure)**.** *For a given combination of non-deposit coupon c and deposit coupon d, the regulator closes the bank at the time T_b when the bank's cash-flow x_t first crosses the following closure trigger x_b:*

$$x_b = \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c + d}{\rho} \tag{9}$$

The closure trigger x_b declines with the growth rate of assets μ and standard deviation σ. The regulator is more lenient with high growth and high uncertainty banks since these institutions are more likely to recover from a near bankruptcy scenario. Banks with negative growth and low uncertainty will, almost surely, continue to shrink cash-flows as time passes and the regulator is therefore less likely to wait for them to recover. Furthermore, a regulator facing a higher discount rate ρ selects a higher trigger x_b. In this case, the regulator becomes more impatient[20] and closes the bank sooner.

A higher regulatory rate ξ increases the closure trigger. This reflects a tighter capital regulation since it enlarges the percentage by which the value of assets should exceed the accounting value of liabilities. Higher non-deposit coupon c and deposit coupon d also increase the closure trigger because they reduce the value of assets available after debt payments.

Figure 3 presents an example that illustrates value functions $O(x_t)$, $R(x_t)$ and the closure trigger x_b. In this example, the regulator decides to close the bank when cash flows hit the trigger $x_b = 10.45$. When the closure option is exercised at this cash flow level, the regulator's flow of utility $R(x_t)$ from the bank climbs to zero.

3.3 The Bank's Problem

This subsection derives the bank's optimisation problem. To do so, I start by deriving the bank's value function. I further present the No Bank-Run condition necessary to enforce Assumption 4.

3.3.1 Value Functions

The bank's value function is given by the following accounting identity:

$$B(x_t) := E(x_t) + C(x_t) + D \quad , \forall t \in [0, T_b)$$

[19] The option elasticity can be written as

$$\frac{dO}{O(x)} \frac{x}{dx} = \beta$$

For example, for a 1% increase in cash-flows x_t, there is a decrease in the closure option's value equal to β%.

[20] Cardoso and Pereira (2015) and Grenadier and Wang (2005) study how different degrees of impatience between a principal and an agent affect the exercise of investment options.

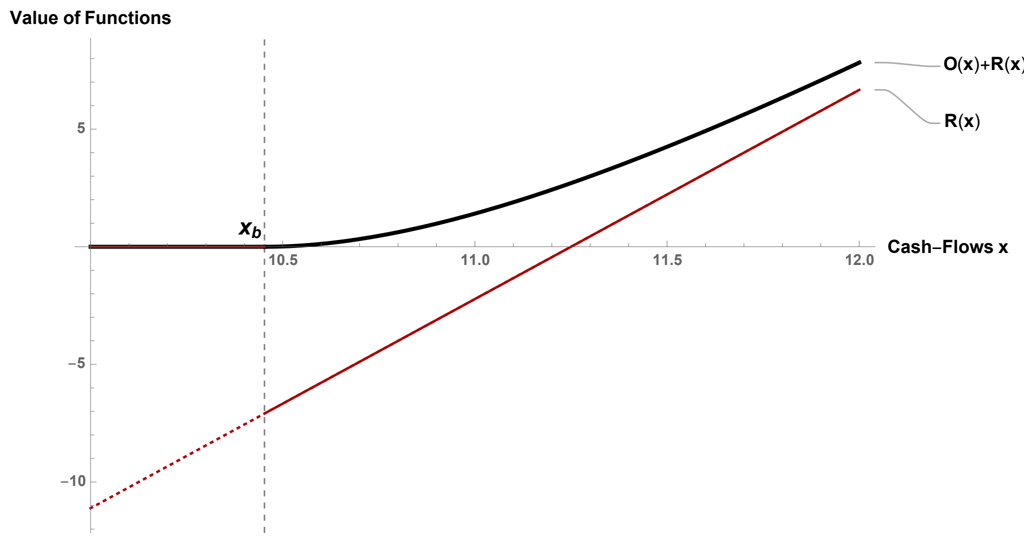


Figure 3: ***The Decision to Close the Bank.*** *The regulator experiences positive utility for as long as $R(x)$ is positive. Capital regulation is violated when $R(x)$ becomes negative (x drops below 11.25) but the regulator will not close the bank immediately. Only when cash-flows deteriorate to the point of crossing trigger $x_b = 10.45$ will the regulator shut down the institution.* ***Parameters:*** $\rho = .1$, $\mu = .01$, $\sigma = .05$, $\xi = .2$.

I therefore start by determining each of the value functions in this identity.

Shareholders For $t \in [0, T_b)$, the shareholders' value is given by the following expression:

$$E(x_t) := \mathbb{E}_t\left[\int_t^{T_b} e^{-\rho(s-t)}(1-\tau)\,(x_s - c - d)\,ds\right] + \\ +\mathbb{E}_t\left[\int_{T_b}^{+\infty} e^{-\rho(s-t)}\max\Big\{0,(1-\tau)\,((1-\epsilon)x_b - c - d)\,\Big\}ds\right] \tag{10}$$

Shareholders receive periodic cash-flows after interest payments and taxes $(1-\tau)(x_t - c - d)$ while the financial institution is operational.

When the bank is closed, assets are sold to another institution, where they will continue to generate cash-flows. The impaired asset value obtained from this sale is distributed among creditors first and only then shareholders given their status of residual claimants.

Shareholders are therefore entitled to the present value of $(1-\tau)((1-\epsilon)x_b - c - d)$ from time T_b until infinity as represented in function (10). Furthermore, shareholders are protected by limited liability. It follows that their income from the bank's assets is never negative and, hence, that they receive the maximum of the after-tax impaired cash flows and zero, as in Equation (10).

Using standard procedures described in Dixit and Pindyck (1994), the shareholders' value function $E(x_t)$ can be written as follows:

$$E(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c+d}{\rho}\right) - \left(\frac{x_t}{x_b}\right)^{\beta}(1-\tau)\min\left\{\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}, \epsilon\frac{x_b}{\rho-\mu}\right\} & , \text{ for } x_t > x_b \\ (1-\tau)\max\left\{0,(1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right\} & , \text{ for } x_t \leq x_b \end{cases} \tag{11}$$

The value function (11) has several components. When the bank is operational ($x_t > x_b$), the equity value is the sum of two terms: the first term, $(1-\tau)(x_t/(\rho-\mu)-(c+d)/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to creditors; the second term is the expected change in value if the regulator closes the bank.

The expected change in equity value is the product of the expected discount factor $(x_t/x_b)^\beta$ and the change in equity value if the institution is closed. This last term is the difference between the closure equity value $(1-\tau)\max\{0,(1-\epsilon)x_b/(\rho-\mu)-(c+d)/\rho\}$ and the after tax present value of cash-flows net of debt payments $(1-\tau)\left(x_t/(\rho-\mu)-(c+d)/\rho\right)$.

Because the shareholders have limited liability, their payoff when the bank is closed ($x_t \leq x_b$) is at least zero. They receive a zero payoff when the impaired asset value $(1-\epsilon)V(x_b)$ is insufficient to cover the present value of coupons L. If the impaired asset value covers the present value of coupons, shareholders obtain the residual value after paying creditors and paying taxes[21].

Non-Deposit Creditors The value function for non-deposit creditors is as follows:

$$C(x_t) := \mathbb{E}_t\left[\int_t^{T_b} e^{-\rho(s-t)}c ds + \int_{T_b}^{+\infty} e^{-\rho(s-t)}\min\Big\{c,(1-\epsilon)x_b-d\Big\}ds\right] \tag{12}$$

Non-deposit creditors earn a periodic coupon c while the bank is alive. When the regulator closes the bank at time T_b, these creditors receive the full amount of the present value of future coupons if the impaired asset value is sufficient to cover their claim after paying depositors.

However, if the impaired asset value is lower than future payments to all creditors, non-deposit creditors suffer a haircut. They only get the impaired asset value after paying depositors, that is, the present value of $((1-\epsilon)x_b-d)$ from time T_b until infinity.

Recall that the closure time T_b is the first time at which the cash flow x_t crosses the trigger x_b. Using this fact, I re-write the value function (12) as follows:

$$C(x_t) = \begin{cases} \frac{c}{\rho} + \left(\frac{x_t}{x_b}\right)^\beta \min\Big\{0,(1-\epsilon)\frac{x_b}{\rho-\mu}-\frac{c+d}{\rho}\Big\} & \text{, for } x_t > x_b \\ \min\Big\{\frac{c}{\rho},(1-\epsilon)\frac{x_b}{\rho-\mu}-\frac{d}{\rho}\Big\} & \text{, for } x_t \leq x_b \end{cases} \tag{13}$$

While the bank is alive ($x_t > x_b$), the value of non-deposit credit comprises two terms: the first term, c/ρ, is the present value of coupons; the second term is the expected change in the non-deposit credit value function if the bank closes.

The expected change in value is the product between the expected discount factor $(x_t/x_b)^\beta$ and the change in creditors' value function at closure. Creditors lose c/ρ when the institution is closed and receive in exchange the closure payoff $\min\{c/\rho,(1-\epsilon)x_b/(\rho-\mu)-d/\rho\}$.

When the institution closes ($x_t \leq x_b$), non-deposit creditors are fully paid if the impaired asset value covers all creditors, in which case, they receive the present value of coupons c/ρ. If the impaired asset value is too low, non-deposit creditors take a loss by receiving the impaired asset value net of deposits $(1-\epsilon)x_b/(\rho-\mu)-d/\rho$.

Depositors By Assumption 3, the bank selects its capital structure to ensure that runs cannot occur. Consequently, the depositor value function is invariant to the state variable x_t. It can be written as follows:

$$D = \int_t^{+\infty} e^{-\rho(s-t)}d\ dt = \frac{d}{\rho} \tag{14}$$

[21] I account for taxation over this residual value as it constitutes an asset sale profit for shareholders.

Total Bank Value Summing the claims of shareholders, non-deposit creditors and depositors, I obtain the bank's value function.

$$B(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_t}{x_b}\right)^{\beta}\left(B_{\mathcal{C}}(x_b) - (1-\tau)\frac{x_b}{\rho-\mu} - \tau\frac{c+d}{\rho}\right) & \text{, for } x_t > x_b \\ \\ B_{\mathcal{C}}(x_b) & \text{, for } x_t \leq x_b \end{cases} \tag{15}$$

Here $B_{\mathcal{C}}(x_b)$ is the value of the bank when it is closed:

$$B_{\mathcal{C}}(x_b) := \frac{d}{\rho} + \min\left\{\frac{c}{\rho}, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{d}{\rho}\right\} + (1-\tau)\max\left\{0, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right\}$$

So long as the bank is alive ($x_t > x_b$), its total value is the sum of three components: the present value of cash-flows after taxes, the debt tax shield, and the expected change to the bank's value given the regulator's closure option.

The expected change of value is the product of the expected discount factor $(x_t/x_b)^{\beta}$ and the change in the bank's value upon closure: that change is equal to the difference between the closure value $B_{\mathcal{C}}(x_b)$ and the after-tax present value of cash flows net of interest payments.

At closure, the bank value $B_{\mathcal{C}}(x_b)$ comprises the certain deposit value added to the value at closure of the claims of non-deposit creditors and shareholders.

I define a *strong* bank to be one whose asset value upon closure, net of bankruptcy costs, is sufficient to cover the value of the coupon payments promised to all creditors. That is, a strong bank satisfies the following condition:

$$(1-\epsilon)V(x_b) \geq L$$

I define a *weak* bank to be any bank that does not satisfy this condition. Lemma 1 demonstrates that the strength of a bank is determined by the capital adequacy regulatory ratio, ξ and, hence, that it can be selected by the regulator. Specifically, there is a boundary value for ξ that separates strong from weak banks:

Lemma 1 (Bank's Type Boundary). *Let*

$$\overline{\xi} = \frac{\beta\epsilon - 1}{\beta - 1} \tag{16}$$

Then any bank whose capital regulation $\xi \geq \overline{\xi}$ is strong; any bank for which $\xi < \overline{\xi}$ is weak.

The boundary $\overline{\xi}$ is the regulatory ratio that sets the bankruptcy asset value equal to the present value of coupons at the bank's closure. Banks with tighter capital requirements are closed sooner and, hence, they are better able to meet their liabilities.

Figure 4 illustrates this feature of my model. In this example, the bank's type boundary is $\overline{\xi} = 0.4$. For any regulatory requirement ratio $\xi \in (0, 0.4)$, the impaired asset value $(1-\epsilon)V(x_b)$ upon bank closure is insufficient to support the value of coupon payments L promised to all creditors. For a $\xi \in (0.4, 1)$, the bank generates an impaired asset value at closure that always covers all creditors.

3.3.2 Preventing Bank Runs

I now discuss my treatment of deposit runs. Assumptions 2 and 3 establish the behaviour of banks when the financial system lacks deposit insurance. With these assumptions, and using the results obtained in Proposition 1, I can derive the "No Bank-Run" condition of this model; the condition imposes a restriction on the optimal mix of deposits and non-deposit coupons.

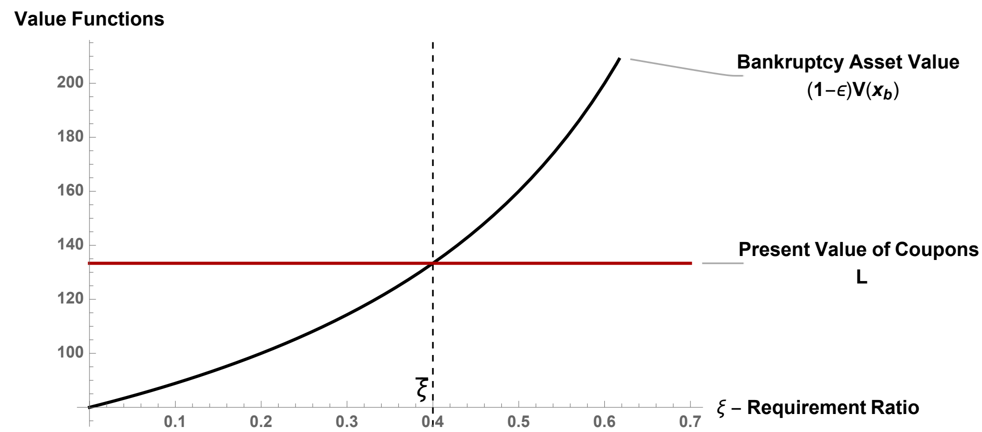


Figure 4: ***The type of bank. Fixed Parameters:*** *$\mu = .01$, $\sigma = 0.1$; $\rho = 0.15$; $\epsilon = 0.3$; $c = 10$; $d = 10$. $\bar{\xi} = 0.4$. For some exogenous regulatory ratio ξ located on the left of $\bar{\xi}$, the bank will be considered weak since the impaired asset value of assets is lower than the present value of interest payments. If $\xi \geq \bar{\xi}$, the bank is considered strong.*

Lemma 2 (No Bank-Run Condition)**.** *For any feasible pair of coupons (c, d), depositor bank runs never occur if and only if condition (17) is satisfied:*

$$\frac{c}{d} \geq \gamma \equiv \frac{\beta - 1}{\beta} \frac{1 - \xi}{1 - \epsilon} - 1 \tag{17}$$

Ratio γ in condition (17) is the minimum quotient of non-deposit coupon c to the deposit coupon d that prevents a run by depositors. By respecting $c/d \geq \gamma$, the bank secures the impaired asset value at closure $(1 - \epsilon)V(x_b)$ exceeds the present value of deposit coupons D. This is sufficient for two reasons: first, depositors are senior over non-deposit creditors when the impaired asset value is distributed among claimants; second, the closure trigger x_b is the lowest level of cash-flows attained during the bank's operation and so, the impaired asset value always covers depositors for any level $x_t > x_b$.

Ratio γ declines with growth rate of assets μ, increases with standard deviation σ and increases with bankruptcy costs fraction ϵ. A bank with high growth, low volatility of assets and low bankruptcy costs has a high impaired asset value $(1 - \epsilon)V(x_b)$. More value is therefore available to cushion deposits at closure leading to a low γ ratio. This institution has more flexibility to select a mix of non-deposit liabilities and deposits that prevents bank runs than a bank with low growth rate μ of assets, high standard deviation σ and high bankruptcy costs fraction ϵ.

The ratio γ decreases if the regulatory requirement ratio ξ increases. This identifies a trade-off between regulatory requirements and bank self-protection. The regulator chooses higher closure triggers for higher regulatory ratio ξ which guarantees that more debt can be covered in bankruptcy. Since depositors are senior, more resources are available for depositors at closure which relaxes the minimum ratio of non-deposit over deposits coupons that prevents runs.

3.3.3 Initial Capital Requirement

Assumption 4 sets a maximum level for the optimal coupons selected at time 0. This assumption prevents the bank from deliberately harming the regulator when defining the capital structure.

Rearranging the regulation condition (4) at $t = 0$, I obtain the Initial Capital Requirement (ICR) condition:

$$c + d \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 \tag{18}$$

3.3.4 Non-Negative Coupons

By Assumption 1, I require each type of liabilities to be non-negative:

$$c \geq 0 \qquad \text{and} \qquad d \geq 0 \tag{19}$$

3.3.5 Bank's Capital Structure Optimisation

The bank's optimal capital structure problem 1 is set up as follows. The bank selects coupons (c, d) to pay creditors that maximise the total bank value $B(x_0)$ while respecting the regulator's optimal closure trigger condition (9), the NBR condition (17), the ICR condition (18), and the non-negative coupons condition (19).

Optimisation Problem 1 (Capital Structure of Base Case)**.** *The bank's optimal capital structure problem is:*

$$\begin{aligned}
(c^*, d^*) \quad & \in \underset{\{c,d\}}{\arg\max}\ B(x_0) & \\
s.t. \quad & x_b := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c+d}{\rho} & \textit{[Closure Trigger]} \\
& \frac{c}{d} \geq \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 & \textit{[No Bank-Run]} \\
& c + d \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 & \textit{[Initial Capital Requirement]} \\
& c \geq 0 \quad , \quad d \geq 0 & \textit{[Non-Negative Coupons]}
\end{aligned}$$

3.4 The Base Case Equilibrium

I now solve Optimisation Problem 1 to determine the equilibrium of the base case. The solution does not pin down unique values[22] for c and d: I solve instead for the total coupon $l = c + d$ and determine the acceptable ranges for c and d.

Proposition 2 (Base Case Equilibrium)**.** *The equilibrium of the game is:*

$$\mathcal{E} = (c^*, d^*, x_b^*)$$

Bank Optimal Decision. *The total periodic coupon $l^* = c^* + d^*$ is optimal iff:*

$$l^* = \min\left\{l^{**}, \bar{l}\right\} \tag{20}$$

where

$$l^{**} = \begin{cases} \frac{(\beta-1)(1-\xi)\rho\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \textit{for } \xi < \bar{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \textit{for } \xi \geq \bar{\xi} \end{cases} \tag{21}$$

[22] Deposits and non-deposit liabilities are indistinguishable when I get the first order condition of the bank's value function (15) with respect to c and d. The differentiating aspect is the NBR condition which allows me to setup the range of minimum optimal non-deposit coupon c^*.

and

$$\bar{l} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{22}$$

Any combination of c^* and d^* that respects the following conditions is optimal:

$$d^* := l^* - c^* \qquad \text{and} \qquad c^* \in \begin{cases} (\hat{c}, \overline{c}) & , \quad \text{for} \quad l^{**} \geq \bar{l} \\ (\underline{c}, \tilde{c}) & , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{23}$$

with

$$\hat{c} = \begin{cases} -\frac{\rho(\beta(\epsilon-\xi)+\xi-1)}{(\beta-1)(\mu-\rho)}x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{24}$$

$$\overline{c} = \frac{\rho(1-\xi)}{\rho-\mu}x_0 \tag{25}$$

$$\underline{c} = \begin{cases} \frac{\rho}{\rho-\mu}\frac{\beta(\epsilon-\xi)+\xi-1}{\beta}\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta}x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{26}$$

$$\tilde{c} = \begin{cases} \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta}x_0 & , \quad \xi < \overline{\xi} \\ \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{27}$$

Regulator Optimal Decision. *At time T_b, cash-flows x_t hit the trigger x_b^*, the Regulator steps in and imposes a bankruptcy. That trigger is:*

$$x_b^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c^*+d^*}{\rho} \tag{28}$$

At time 0, the bank selects an optimal total coupon l^* according to condition (20). In detail, the bank selects $l^* = l^{**}$ if the initial capital requirement is not violated. That is the case if $l^{**} < \bar{l}$. Otherwise, the institution's optimal total coupon has the corner solution $l^* = \bar{l}$. In this case, the solution binds at the ICR condition and the regulator has zero utility at $t = 0$.

The bank must also choose a combination of deposits d^* and non-deposit liabilities c^*. This is characterised in condition (23). The sum of deposit and non-deposit coupons must always equal the total coupon l^* as formulated in $d^* := l^* - c^*$ in condition (23).

I use the non-deposit coupon to write the interval of feasible optimal solutions. The upper-bound of the c^* interval is the optimal total coupon l^*. A non-deposit coupon c^* closer to the total coupon l^* requires a deposit coupon d^* closer to zero. The lower-bound of c^* depends on the type of bank. A weak bank selects a c^* that respects the NBR condition to prevent depositors' runs. A strong bank is not limited by the NBR condition and only needs to guarantee that the lower-bound of c^* is always zero to respect the non-negativity constraint. Figure 5 illustrates the interval for the present value of non-deposit liabilities c^*/ρ in equilibrium.

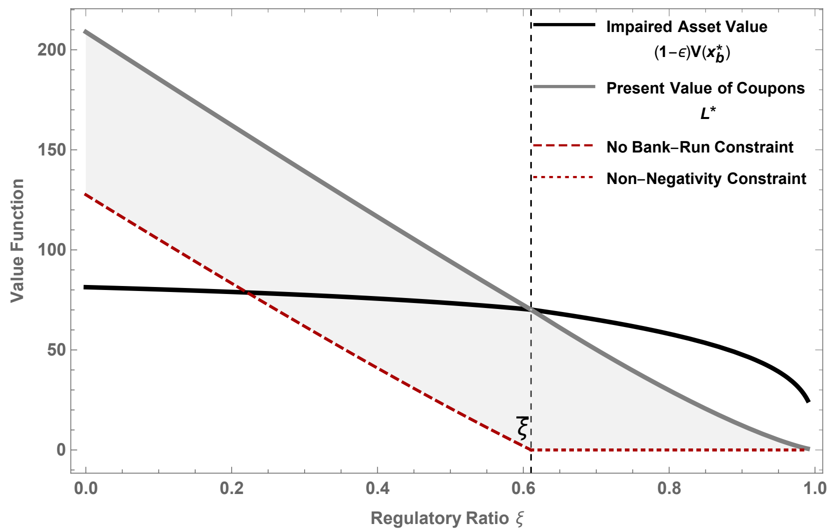


Figure 5: ***Example of*** c^*/ρ ***interval. Parameters:*** $\mu = -.02$, $\sigma = 0.1$; $\rho = 0.15$; $\epsilon = 0.5$. $\bar{\xi} = 0.61$. *The grey area defines the interval for* c^*/ρ. *A strong bank* ($\xi \geq \bar{\xi}$) *has an impaired asset value that covers the present value of coupons for all creditors. The strong bank can choose any level of non-deposit liabilities as long as it is positive.*

A weak bank ($\xi < \bar{\xi}$) *cannot cover the present value of coupons with the impaired asset value. To prevent runs the weak bank should hold at least the minimum level of* c^*/ρ *as represented in the red dashed line. Any coupon* c^* *that satisfies that condition prevents bank runs. Beyond that level, the bank is indifferent between deposit and non-deposit debt finance.*

4 Costless Deposit Insurance

4.1 Elements of the Model

In this section, I add a Deposit Insurance Scheme (DIS) to my framework. I use the subscript "*di*" to distinguish the value functions, parameters and controls in this section.

▷ **The Bank.** The bank's assets generate cash flow x_t that follows the GBM defined in equation (1) with drift $\mu \in (-\infty, \rho)$, volatility $\sigma > 0$ and exogenous discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (2).

The bank selects coupon c_{di} for non-deposit creditors and a coupon d_{di} for depositors at time 0. These coupons are paid until the bank is closed. I write the total coupon as $l_{di} = c_{di} + d_{di}$. The accounting value of liabilities L_{di} is the present value of total future coupon payments:

$$L_{di} = \int_t^{+\infty} e^{-\rho(s-t)}(c_{di} + d_{di})ds = \frac{c_{di} + d_{di}}{\rho}, \tag{29}$$

At closure time, the assets of the failing bank are bought by another institution. As in Section 3.1, the impaired asset value after asset sales is $(1-\epsilon)V(x_t)$ and, also as in Section 3.1, the corporate tax rate τ is levied on cashflows net of interest payments.

The bank's value function can be formulated as the sum of the value of the claims:

$$B_{di}(x_t) = E_{di}(x_t) + C_{di}(x_t) + D_{di}$$

where $E_{di}(x_t)$ is the value of the claim held by the equity-holders, $C_{di}(x_t)$ is the value function for Non-deposit Creditors and D_{di} is the value function for depositors. In line with Section 3.1, I write the capital structure ratio $CS_{di}(x_t)$ and the debt structure ratio $DS_{di}(x_t)$ as:

$$CS_{di}(x_t) := \frac{E_{di}(x_t)}{B_{di}(x_t)} \qquad\qquad DS_{di}(x_t) := \frac{C_{di}(x_t)}{C_{di}(x_t) + D_{di}}$$

▷ **Bank's Closure.** The regulator closes the bank at time T_{di} when cash-flows x_t first cross the trigger x_{di}:

$$T_{di} = \inf\{t \in [0, +\infty) : x_t \notin [x_{di}, +\infty)\} \tag{30}$$

As in Subsection 3.1, the regulator trades off the utility benefit of immediate enforcement against the potential benefits achieved when the bank's cash flow recovers. Following Proposition 1, the closure trigger x_{di} in the DIS case is:

$$x_{di} = \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_{di} + d_{di}}{\rho} \tag{31}$$

where $\xi \in (0, 1)$ is an exogenous regulatory requirement ratio depicted in equation (4) and β is the elasticity of the closure option value to cash-flows outlined in equation (8).

▷ **Deposit Insurance.** Now assume that all depositors are fully protected by a costless deposit insurance scheme[23]. I assume that:

Assumption 5. *The deposit insurance fund never defaults.*

The deposit insurance fund covers the difference between the present value of deposits and the impaired asset value, if the latter is insufficient to pay the depositors. The deposit insurance scheme therefore makes the following transfer to bank depositors upon the bank's closure:

$$\max\left\{0, \frac{d_{di}}{\rho} - (1 - \epsilon)V(x_{di})\right\} \tag{32}$$

Full deposit insurance prevents runs, and so it renders the No Bank-Run condition (17) irrelevant.

▷ **Equilibrium.** As in Section 3, the equilibrium has two stages: stage 1 runs from time 0 until the bank closes at time T_{di}, and stage 2 runs from time T_{di} onwards.

At time 0, the bank selects the non-deposit coupon c_{di} and the deposit coupon d_{di} so as to maximise the value function $B_{di}(x_0)$. When the bank is closed at time T_{di}, its impaired asset value is distributed in line with the claimant hierarchy. The deposit insurance scheme is called upon only if the impaired asset value is insufficient to meet depositor claims.

As in Section 3, I solve for a sequential equilibrium $\mathcal{E}_{di}$:

$$\mathcal{E}_{di} := (c^*_{di}, d^*_{di}, x^*_{di})$$

At $t = 0$, the bank selects the value maximising pair of coupons (c^*_{di}, d^*_{di}). The regulator closes the bank at time T_{di}, when the cash-flow x_t first crosses the closure trigger $x^*_{di} := x_{di}(c^*_{di}, d^*_{di})$.

[23] A natural extension is to account for an insurance fee in proportion of deposits as, for example, seen in the liability structure models of Sundaresan and Wang (2017) and Gambacorta et al. (2017). This fee increases the bank's cost of owning deposits. Banks will consequently be less willing to accept deposits as a source of debt. Morrison and White (2011) suggest however that capitalising the insurance fund through bank contributions is welfare-neutral. A better alternative is instead to use general taxation on agents outside the the financial system prevents banks from crowding-out deposits from its liability structure.

4.2 The Bank's Problem

This subsection presents the bank's optimisation problem under costless deposit insurance. I derive the value function of the bank and explain how the DIS affects the bank's claimants.

4.2.1 Value Functions

The value of the bank satisfies the following accounting identity at all times:

$$B_{di}(x_t) := E_{di}(x_t) + C_{di}(x_t) + D_{di} \quad , \forall t \in [0, T_{di})$$

I therefore start by determining the value functions for each one of these claims.

Shareholders For $t \in [0, T_{di})$, the value function of shareholders can be written as follows:

$$\begin{aligned} E_{di}(x_t) := \ & \mathbb{E}_t \left[\int_t^{T_{di}} e^{-\rho(s-t)} (1-\tau)\left(x_s - c_{di,s} - d_{di,s}\right) ds \right] + \\ & \mathbb{E}_t \left[\int_{T_{di}}^{+\infty} e^{-\rho(s-t)} \max\left\{0, (1-\tau)\left((1-\epsilon)x_{di} - c_{di} - d_{di}\right)\right\} ds \right] \end{aligned} \tag{33}$$

The first term in Equation (33) is the value to equity holders of the cash flows the bank generates while it is operating: at every $t \in [0, T_{di})$ shareholders receive after tax cash-flows net of interest payments $(1-\tau)\left(x_s - c_{di,s} - d_{di,s}\right)$.

The second term in Equation (33) is the value that equity holders derive from the bank when it is closed. The bank's assets are sold upon closure to another institution that receives all of their subsequent cash flows. As payment from the asset sale, the equity holders of the closed bank receive the impaired asset value net of liabilities and taxes. That is, they receive the present value of the cash flow stream $(1-\tau)\left((1-\epsilon)x_{di} - c_{di} - d_{di}\right)$ from time T_{di} until infinity if that value is non-negative; if it is not, then, because they are protected by limited liability, they receive zero.

It is immediate from a comparison of Equation (10) with Equation (33) that the only effect that deposit insurance can have upon the value of equity is through its effect on the closure time T_{di}. That is because deposit insurance pays out only after default and, hence, it does not affect cash flows before time T_{di}. It follows immediately that, by analogy with Equation (11), $E_{di}(x_t)$ can be written as follows:

$$E_{di}(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right) - \left(\frac{x_t}{x_{di}}\right)^{\beta} (1-\tau) \min\left\{\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}, \epsilon\frac{x_{di}}{\rho-\mu}\right\} & , \text{for } x_t > x_{di} \\ (1-\tau) \max\left\{0, (1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right\} & , \text{for } x_t \leq x_{di} \end{cases} \tag{34}$$

The intuition for Equation (34) is identical to the intuition for Equation (11).

Non-Deposit Creditors Non-deposit creditors earn no direct benefit from a deposit insurance scheme. However, these creditors experience an indirect effect because deposit insurance alters the closure time T_{di} and, hence, affects the closure payoff. In particular, if closure is delayed so long that the impaired asset value at time T_{di} is below the present value of deposits, non-deposit creditors may receive a zero payoff upon closure.

The following expression for the value $C_{di}(x_t)$ of their claim follows by analogy with Equation (12)

$$C_{di}(x_t) := \mathbb{E}_t\left[\int_t^{T_{di}} e^{-\rho(s-t)} c_{di,s} ds + \int_{T_{di}}^{+\infty} e^{-\rho(s-t)} \min\left\{c_{di}\ , \max\left\{(1-\epsilon)x_{di} - d_{di}\ , 0\right\}\right\} ds\right] \tag{35}$$

Equation (35) follows the same intuition of Equation (12) with the sole difference that, with deposit insurance, non-deposit creditors can get a zero payoff at closure. The reason that this could happen with deposit insurance is as follows. Deposit insurance releases the bank from having a capital structure that prevents bank runs and, hence, it can select pairs c_{di} and d_{di} for which the impaired asset value is lower than the present value of cash-flows: that is, for which $(1-\epsilon)V(x_{di}) < d_{di}/\rho$. If this happens, then, because non-deposit creditors are junior claimants, they receive a zero payoff.

Again by analogy with Section 3, the value function in Equation (35) can be written as follows:

$$C_{di}(x_t) = \begin{cases} \frac{c_{di}}{\rho} + \left(\frac{x_t}{x_{di}}\right)^\beta \min\left\{0\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\ , -\frac{c_{di}}{\rho}\right\}\right\} & , \text{ for } x_t > x_{di} \\ \min\left\{\frac{c_{di}}{\rho}\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\ , 0\right\}\right\} & , \text{ for } x_t \leq x_{di} \end{cases} \tag{36}$$

The intuition for Equation (36) is identical to the intuition of Equation (13).

Depositors Deposit insurance protects depositors from losses when the impaired asset value $(1-\epsilon)V(x_{di})$ is lower than the present value of future deposit payments d_{di}/ρ. Therefore, the value of deposits does not depend upon the cash flow process x_t:

$$D_{di} = \mathbb{E}_t\left[\int_t^{+\infty} e^{-\rho(s-t)} d_{di} dt\right] = \frac{d_{di}}{\rho} \tag{37}$$

Total Bank Value Summing the claims of shareholders, non-protected creditors and depositors, I obtain the following value function for the bank in the presence of costless deposit insurance.

$$B_{di}(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{di}+d_{di}}{\rho} + \left(\frac{x_t}{x_{di}}\right)^\beta \left(B_{di,\mathcal{C}}(x_{di}) - (1-\tau)\frac{x_{di}}{\rho-\mu} - \tau\frac{c_{di}+d_{di}}{\rho}\right) & , \text{ for } x_t > x_{di} \\ B_{di,\mathcal{C}}(x_{di}) & , \text{ for } x_t \leq x_{di} \end{cases} \tag{38}$$

Here $B_{di,\mathcal{C}}(x_{di})$ is the bank's closure value:

$$\begin{aligned} B_{di,\mathcal{C}}(x_{di}) := \frac{d_{di}}{\rho} + \min\left\{\frac{c_{di}}{\rho}\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\ , 0\right\}\right\} \\ +(1-\tau)\max\left\{0, (1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right\} \end{aligned}$$

While the bank is open $(x_t > x_{di})$, its value function is the sum of the present value of cash-flows after taxes, the debt tax shield and the expected change in value when the closure option is exercised. That change is the product of the expected discount factor $(x_t/x_{di})^\beta$ at the closure date and the difference between the bank's closure value $B_{di,\mathcal{C}}(x_{di})$ and the combined present value of cashflows $(1-\tau)x_{di}/(\rho-\mu)$ and the tax rebate $\tau(c_{di}+d_{di})/\rho$.

The three terms in the bank's closure value $B_{di,\mathcal{C}}(x_{di})$ are, respectively, the present value of payments to depositors, the non-deposit credit value at closure and the equity value at closure.

In Section 3.3.1, I defined a strong bank to be one whose impaired asset value is guaranteed to be sufficient to cover the present value of all future interest payments at closure $(1-\epsilon)V(x_{di}) \geq L_{di}$. A weak bank is one that does not satisfy this condition. Because banks no longer have to satisfy a no-run condition with a DIS, it is useful to distinguish between banks that do, and do not, need to call upon the DIS upon failure.

I define an *independent* bank to be a weak bank whose impaired asset value is sufficient to cover the present value of coupons promised to depositors; independent banks therefore do not draw upon the deposit insurance fund. An independent bank satisfies the following condition:

$$(1-\epsilon)V(x_b) \geq D_{di} \tag{39}$$

I define a *dependent* bank to be a weak bank that violates Condition (39). Dependent banks are obliged to draw upon the deposit insurance fund when they are closed. Lemma 3 relates the capital adequacy of a bank to its dependency upon the deposit insurance fund.

Lemma 3 (DIS Dependence Boundary)**.** *Let*

$$\underline{\xi} := \underline{\xi}(c_{di}, d_{di}) = \frac{c_{di}\beta(1-\epsilon) + d_{di}(1-\beta\epsilon)}{d_{di}(1-\beta)} \tag{40}$$

Then any weak bank whose capital regulation $\xi \geq \underline{\xi}$ is independent; any bank for which $\xi < \underline{\xi}$ is dependent.

The boundary $\underline{\xi}$ is the regulatory ratio that sets the bankruptcy asset value equal to the present value of deposits at the bank's closure. Banks with tighter capital requirements are closed earlier and, hence, are better able to meet depositors payments.

Figure 6 illustrates the boundaries in the DIS case. Unlike the $\overline{\xi}$ boundary, $\underline{\xi}$ depends on the composition of debt. In Subfigure 6.a the weak bank has both types of debt and so, depending on the exogenous regulatory ratio ξ, it may be DIS dependent or independent. In Subfigure 6.b, the weak bank uses deposits as the single source of debt and, consequently, it will need the intervention of the deposit insurance fund at the bank's closure. In this situation, $\underline{\xi} = \overline{\xi}$. Finally, in Subfigure 6.c, the weak bank only uses non-deposit liabilities and so, when it closes, the non-deposit creditors will fully absorb the bankruptcy losses. In this situation, $\underline{\xi} = 0$.

Since $\underline{\xi}$ depends upon coupon levels c_{di} and d_{di}, it is determined in equilibrium. I write the equilibrium DIS dependence boundary as $\underline{\xi}^* = \underline{\xi}(c^*_{di}, d^*_{di})$.

4.2.2 Initial Capital Requirement

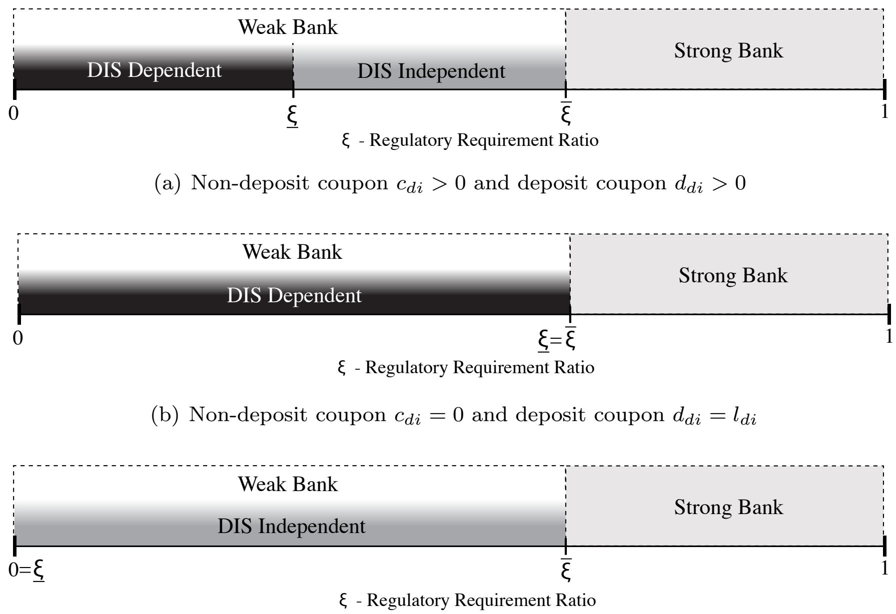
I continue to impose Assumption 4: that is, that the bank respects the capital requirement ξ when it selects its initial capital structure. The Assumption yields the following Initial Capital Requirement:

$$c_{di} + d_{di} \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{41}$$

4.2.3 Non-Negative Coupons

By Assumption 1, each type of liability must be non-negative:

$$c_{di} \geq 0 \qquad \text{and} \qquad d_{di} \geq 0 \tag{42}$$



(a) Non-deposit coupon $c_{di} > 0$ and deposit coupon $d_{di} > 0$

(b) Non-deposit coupon $c_{di} = 0$ and deposit coupon $d_{di} = l_{di}$

(c) Non-deposit coupon $c_{di} = l_{di}$ and deposit coupon $d_{di} = 0$

Figure 6: *Weak bank boundaries in the presence of costless deposit insurance.*

4.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_{di}, d_{di}) so as to maximise the total bank value $B_{di}(x_0)$ subject to the regulator's optimal closure trigger in condition (31), the Initial Capital Requirement condition (41) and the non-negative coupon condition (42).

Optimisation Problem 2 (Capital Structure with deposit insurance)**.** *The bank's optimal capital structure problem is:*

$$
\begin{aligned}
(c^*_{di}, d^*_{di}) \quad & \in \arg\max_{\{c_{di}, d_{di}\}} B_{di}(x_0) \\
s.t. \quad & x_{di} := \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_{di} + d_{di}}{\rho} && \textit{[Closure Trigger]} \\
& c_{di} + d_{di} \leq \frac{\rho(1 - \xi)}{\rho - \mu} x_0 && \textit{[Initial Capital Requirement]} \\
& c_{di} \geq 0 \quad , \quad d_{di} \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}
$$

4.3 The Deposit Insurance Equilibrium

Proposition 3 (Costless Deposit Insurance Case Equilibrium)**.** *The equilibrium of the game with costless deposit insurance is:*

$$\mathcal{E}_{di} = (c^*_{di}, d^*_{di}, x^*_{di}) \tag{43}$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}$$

Bank Optimal Decision. *The total periodic coupon* $l_{di}^* = c_{di}^* + d_{di}^*$ *is optimal iff:*

$$l_{di}^* = \min\left\{l_{di}^{**}, \bar{l}_{di}\right\} \tag{44}$$

where

$$l_{di}^{**} = \begin{cases} \frac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0, & \text{for } \xi < \bar{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)}x_0, & \text{for } \xi \geq \bar{\xi} \end{cases} \tag{45}$$

and

$$\bar{l}_{di} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{46}$$

Any combination of c_{di}^* *and* d_{di}^* *that respects the following conditions is optimal:*

$$d_{di}^* := l_{di}^* - c_{di}^* \qquad \text{and} \qquad c_{di}^* = \begin{cases} 0 & , \quad \xi < \bar{\xi} \\ c_{di}^{**} & , \quad \xi \geq \bar{\xi} \end{cases} \tag{47}$$

with

$$c_{di}^{**} \in \begin{cases} \left(0, \frac{\rho(1-\xi)}{\rho-\mu}x_0\right) & , \quad \text{for} \quad l^{**} \geq \bar{l} \\ \\ \left(0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0\right) & , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{48}$$

Regulator Optimal Decision. *At time* T_{di}*, cash-flows* x_t *hit the trigger* x_{di}^**, the Regulator steps in and closes the bank. That trigger is:*

$$x_{di}^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_{di}^* + d_{di}^*}{\rho} \tag{49}$$

The bank chooses total coupon l_{di}^* according to condition (44). It selects the interior solution $l_{di}^* = l_{di}^{**}$ if the initial capital requirement is respected. Otherwise, the bank must select the total coupon $l^* = \bar{l}$ that enforces a regulatory utility of zero at $t = 0$.

The bank selects an optimal combination of deposits d^* and non-deposit liabilities c^* in accordance to condition (47). To guarantee that the sum of c_{di}^* and d_{di}^* respects the equilibrium total coupon l^* we set $d^* := l^* - c^*$.

Weak banks always select $c_{di}^* = 0$ when they have access to costless deposit insurance. This means that the equilibrium dependence boundary $\underline{\xi}^*$ equals the bank's type boundary $\bar{\xi}$ and, so, weak banks choose to become DIS dependent and crowd-out all non-deposit debt. So, in equilibrium, weak banks choose the case presented in Figure 6.b.

A strong bank selects the optimal non-deposit coupons c_{di}^* from the interval of admissible c_{di}^* defined in condition (48). The strong bank must always select a positive non-deposit coupon and, at most, should select a coupon close but lower than the total coupon l_{di}^* which implicitly secures that the regulatory utility is never negative when the bank chooses the capital structure.

4.4 Discussion on the effects of introducing a DIS

I discuss the impact of introducing costless deposit insurance in the equilibrium by comparing the DIS equilibrium in Proposition 3 against the base case equilibrium in Proposition 2. I write the equilibrium capital structure at time 0 for the base case as $CS^*(x_0) \equiv CS(x_0|\mathcal{E})$ and for the DIS case as $CS^*_{di}(x_0) \equiv CS_{di}(x_0|\mathcal{E}_{di})$. I also write the equilibrium debt structure at time 0 for the base case as $DS^*(x_0) \equiv DS(x_0|\mathcal{E})$ and for the DIS case as $DS^*_{di}(x_0) \equiv DS_{di}(x_0|\mathcal{E}_{di})$.

Proposition 4 (Change in the Equilibrium Total Coupon)**.** *Introducing costless deposit insurance weakly increases the equilibrium total coupon paid by the bank:*

$$l^*_{di} \geq l^* \tag{50}$$

Proposition 5 (Change in the Equilibrium Debt Structure)**.** *Introducing costless deposit insurance weakly decreases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS^*_{di}(x_0) \leq DS^*(x_0) \tag{51}$$

Proposition 6 (Change in the Equilibrium Capital Structure)**.** *Introducing costless deposit insurance weakly decreases the equilibrium ratio between equity value and total bank value:*

$$CS^*_{di}(x_0) \leq CS^*(x_0) \tag{52}$$

Propositions 4 to 6 state that, in equilibrium, banks will neither decrease leverage nor decrease deposits if the regulator introduces costless deposit insurance. Precisely what the bank does depends upon its type.

▷ **The Effect of DIS in Weak Banks.** By Lemma 3, the weak bank can be DIS-dependent or independent depending on the combination of c_{di} and d_{di} it selects. In equilibrium, it takes advantage of costless deposit insurance and chooses to be DIS dependent.

With a costless DIS, deposits become more attractive than non-deposits: deposits are covered by the DIS while non-deposit liabilities experience losses at closure (because $(1-\epsilon)V(x_b) < c_{di}/\rho$) if the institution is weak. Therefore, in equilibrium, the weak bank choses to use deposits as its sole debt source:

$$DS^*_{di}(x_0|\xi < \overline{\xi}) = 0 < DS^*(x_0|\xi < \overline{\xi})$$

Then, because it has only deposits in its liability structure, the bank increases its equilibrium leverage since the costs of closure are now born by the insurance fund and the institution can therefore take greater advantage of the debt tax shield. The equilibrium total coupon l^*_{di} is therefore weakly higher than the base case total coupon l^*, and the bank's equity capital level weakly decreases:

$$CS^*_{di}(x_0|\xi < \overline{\xi}) \leq CS^*(x_0|\xi < \overline{\xi})$$

The lower equilibrium equity/bank value ratio reflects a higher debt value and a lower equity value relative to the case without a DIS. The increase in debt is caused by the higher coupon l^*_{di}. Higher leverage lowers the expected value of equity by decreasing the present value of cash-flows after interest payments $(1-\tau)(V(x_t) - (c_{di} + d_{di})/\rho)$, and by increasing the closure trigger $(\partial x_{di}/\partial l_{di} > 0)$ so that the bank is closed earlier.

▷ **The Effect of DIS in Strong Banks.** Strong banks do not change their funding decision when the regulator introduces costless deposit insurance. This is because these institutions do not trigger a payout by the deposit insurance fund. If a strong bank is closed, the impaired asset value always covers the promised future payments to all creditors and, so, its expected payoffs stay unchanged.

A strong bank is unaffected by the introduction of costless deposit insurance; its optimal total debt coupon l^*_{di} is the same as the base case total coupon l^*. Then, as in the proofs of Propositions 5 and 6, the capital structure ratio and the debt structure ratio are unchanged:

$$CS^*_{di}(x_0|\xi \geq \bar{\xi}) = CS^*(x_0|\xi \geq \bar{\xi}) \qquad DS^*_{di}(x_0|\xi \geq \bar{\xi}) = DS^*(x_0|\xi \geq \bar{\xi})$$

The following corollary follows from Propositions 4 to 6:

Corollary 6.1. *Costless deposit insurance incentivises weak financial institutions to use deposits as the exclusive source of debt and to increase total leverage at the expense of the insurance fund. Higher leverage decreases equity value and, hence, the capital structure ratio of weak banks decreases. The incentives of strong bank are not affected by costless deposit insurance.*

▷ **Implementing the change in the Capital Structure.** I conclude this Section by examining the way in which the bank implements the new capital structure when deposit insurance is introduced, and with an examination of the wealth effects of the introduction. The executive team of the bank will never make a capital structure decision that renders the existing shareholders worse-off.

When deposit insurance is introduced, the bank's executive team buys back equity, and pays for it with a partial claim over deposits. The value of this claim over deposits should preserve the existing shareholders' value. I use η_{di} to represent the share of deposits held by existing shareholders after the capital structure changes. We must have:

$$E_{di}(x_0|\mathcal{E}_{di}) + \eta_{di} D_{di}(\mathcal{E}_{di}) \geq E(x_0|\mathcal{E})$$

That is, the final value of the existing shareholder's claim after the change in capital structure should be at least as high as equity value before the DIS is introduced.

Moreover, the value of the shareholders's claim over deposits must be bounded by the bank value increase that results from the change in the capital structure. That is:

$$\eta_{di} D_{di}(\mathcal{E}_{di}) \leq B_{di}(x_0|\mathcal{E}_{di}) - B(x_0|\mathcal{E})$$

Using these two inequalities, I can find the admissible range of η_{di} that maintains existing shareholders satisfied with the capital structure change in response to the introduction of a DIS.

$$\eta_{di} \in \left[\frac{E(x_0|\mathcal{E}) - E_{di}(x_0|\mathcal{E}_{di})}{D_{di}(\mathcal{E}_{di})}, \frac{B_{di}(x_0|\mathcal{E}_{di}) - B(x_0|\mathcal{E})}{D_{di}(\mathcal{E}_{di})}\right] \tag{53}$$

In summary, the introduction of a costless deposit insurance changes the incentives of weak financial institutions but not strong institutions. Weak banks use deposits as their exclusive source of debt because they can transfer closure costs to the deposit insurance fund at no expense (insurance fees). Hence, weak banks choose to be DIS-dependent. At the same time, this gives an incentive to leverage up the institution. However, this leverage increase causes a decrease in the value of equity. Existing shareholders will see the value of the claim decrease because higher debt coupons lower the expected residual claim over cash-flows and increase the intervention trigger. Nonetheless, the capital structure is designed by the bank and, hence, existing shareholders should be compensated for the loss of expected value. This is attained through a buyback of shares in exchange for a claim over deposits.

5 Private Sector Acquisition

5.1 Elements of the Model

The post-financial crisis European regulation grants authorities the right to apply resolution tools to failing financial institutions. In this section, I add the Private Sector Acquisition (PSA) resolution tool to my framework. I use the subscript "p" to distinguish the value functions, parameters and controls in this section.

▷ **The Bank.** The bank's assets generate cash flow x_t following the GBM defined in equation (1) with drift $\mu \in (-\infty, \rho)$, standard deviation $\sigma > 0$ and exogenous discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (2).

The bank selects coupon c_p for non-deposit creditors and a coupon d_p for depositors at time 0. These coupons are paid until the bank is closed. The total coupon is $l_p = c_p + d_p$. The accounting value of liabilities L_p is the present value of total future coupon payments:

$$L_p = \int_t^{+\infty} e^{-\rho(s-t)}(c_{di} + d_{di})ds = \frac{c_{di} + d_{di}}{\rho}, \tag{54}$$

Under the PSA, the sale of assets generates costs of magnitude $\epsilon_p V(x_t)$, with $\epsilon_p \in (1, 0)$. The *impaired asset value* is written as $(1 - \epsilon_p)V(x_t)$. The corporate tax rate $\tau \in (0, 1)$ is levied on cash-flows net of interest payments $(1 - \tau)(x_t - c_{di} - d_{di})$.

The bank selects coupons (c_p, d_p) so as to maximise its continuation value function $B_p(x_t)$:

$$B_p(x_t) = E_p(x_t) + C_p(x_t) + D_p$$

where $E_p(x_t)$ is the value of equity under a PSA, $C_p(x_t)$ is the value function for Non-deposit Creditors and D_p is the value function for depositors. Following Section 3, I write the capital structure ratio $CS_p(x_t)$ as the ratio between equity value and bank value and the debt structure ratio $DS_{di}(x_t)$ as the ratio between the non-deposit creditors value and total liabilities value:

$$CS_p(x_t) := \frac{E_p(x_t)}{B_p(x_t)} \qquad\qquad DS_p(x) := \frac{C_p(x_t)}{C_p(x_t) + D_{p,t}}$$

▷ **Bank's Closure.** Following Subsection 3.1, the bank is closed at time T_p, when cash-flows x_t first cross the trigger x_p:

$$T_p = \inf\{t \in [0, +\infty) : x_t \notin [x_p, +\infty)\} \tag{55}$$

Similarly to Proposition 1, the closure trigger x_p in the PSA case is:

$$x_p = \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_p + d_p}{\rho} \tag{56}$$

where $\xi \in (0, 1)$ is the exogenous regulatory requirement ratio in equation (4) and β is the elasticity of the closure option value to cash-flows in equation (8).

▷ **Private Sector Acquisitions.** Authorities use the Private Sector Acquisition (PSA) tool to intermediate the sale of the assets of a failing bank to other financial institution, regardless of stakeholders' approval[24]. The only way that the PSA changes my model is through its effect upon the closure costs:

[24] For example, under the European Central Bank supervision, this operation comprehends assets, liabilities or even shares (See Art. 38 and 39 of BRRD and Art. 24 of SRM for further details) whereas, for example, under the supervision of the Bank of England, the equivalent tool is restricted to assets.

Assumption 6. *Closure costs are always lower under the PSA. That is:*

$$\epsilon > \epsilon_p$$

The PSA tool decreases closure costs because a resolution authority can avoid externalities that occur in private-sector resolutions (i.e., in bankruptcies managed by creditors). For example, the PSA avoids "rush to the courthouse" and "hold out" externalities (Roubini 2004) that would cause high fire sales costs. The "rush to the courthouse" externality occurs when a group of creditors initiate a lawsuit and attach some of the assets to the lawsuit, preventing other creditors from accessing those assets.

The "holdout" (or "free-riding") externality happens when an advantageous resolution plan does not reach the required quorum in a creditors' assembly. In this circumstance, some creditors may want to haste the resolution process by agreeing to pay these holdouts. However, that ultimately creates an incentive for more creditors to hold out as well. This stalls the sale of assets and erodes the chance for an orderly and less costly resolution.

▷ **Type of Equilibrium.** As in Sections 3 and 4, the equilibrium has two stages: stage 1 runs from time 0 until the bank closes at time T_p, and stage 2 runs from time T_p onwards.

The bank selects the optimal coupons (c_p^*, d_p^*) and the regulator then chooses the closure time T_p. I write $x_p^* := x_p(c_p^*, d_p^*)$ for the bank's closure trigger. The equilibrium $\mathcal{E}_p$ of this sequential game is:

$$\mathcal{E}_p := \left(c_p^*, d_p^*, x_p^*\right)$$

5.2 The Bank's Problem

In this subsection, I present the bank's optimisation problem under the private sector acquisition tool. I derive the value function of the bank and explain the impact that this resolution tool has over the bank's claimants.

5.2.1 Value Functions

The value of the bank satisfies the accounting identity:

$$B_p(x_t) := E_p(x_t) + C_p(x_t) + D_p \quad , \forall t \in [0, T_p)$$

The value functions for the three claimants have the same structure as in Section 4. However, the PSA affects the payoffs at closure for some of the claimants.

Shareholders For $t \in [0, T_p)$, the shareholders' value function can be written as follows

$$E_p(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right) - \left(\frac{x_t}{x_p}\right)^{\beta}(1-\tau)\min\left\{\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}, \epsilon_p\frac{x_p}{\rho-\mu}\right\} & , \text{ for } x_t > x_p \\ (1-\tau)\max\left\{0, (1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right\} & , \text{ for } x_t \leq x_p \end{cases} \tag{57}$$

Equation (57) has the same intuition as Equation (34). The change in the closure cost fraction from ϵ to ϵ_p increases the payoffs at closure for the shareholders of strong banks. Shareholders of weak banks are unaffected at closure because, irrespective of the introduction of the PSA tool, their payoff is zero.

Non-Deposit Creditors The value function for non-deposit creditors can be written as follows:

$$C_p(x_t) = \begin{cases} \frac{c_p}{\rho} + \left(\frac{x_t}{x_p}\right)^{\beta} \min\left\{0 \ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho} \ , -\frac{c_p}{\rho}\right\}\right\} & , \text{ for } x_t > x_p \\ \min\left\{\frac{c_p}{\rho} \ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{d_p}{\rho} \ , 0\right\}\right\} & , \text{ for } x_t \leq x_p \end{cases} \tag{58}$$

The components of Equation (58) have a similar interpretation to those of Equation (36). The decrease in closure costs only affects the closure payoffs for non-deposit creditors of weak independent banks; in those banks, non-deposit creditors receive the residual value after paying deposits: that is, they receive $(1-\epsilon_p)x_p/(\rho-\mu) - d_p/\rho$.

If the bank is not a weak-independent bank, non-deposit creditors do not suffer changes in their closure payoffs: creditors of strong banks receive their full promised claim c_p/ρ, while creditors of weak-dependent banks have a payoff of zero since impaired asset value is entirely used to pay depositors.

Depositors The value function of depositors is unaffected by the introduction of a PSA.

$$D_p = \left[\int_t^{+\infty} e^{-\rho(s-t)} d_p dt\right] = \frac{d_p}{\rho}, \qquad \text{for } t \geq 0 \tag{59}$$

Total Bank Value The total value function for the financial institution is:

$$B_p(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_p+d_p}{\rho} + \left(\frac{x_t}{x_p}\right)^{\beta} \left(B_{p,\mathcal{C}}(x_p) - (1-\tau)\frac{x_p}{\rho-\mu} - \tau\frac{c_p+d_p}{\rho}\right) & , \text{ for } x_t > x_{di} \\ B_{p,\mathcal{C}}(x_p) & , \text{ for } x_t \leq x_p \end{cases} \tag{60}$$

where $B_{p,\mathcal{C}}(x_p)$ is the bank's value at the closure stage and is written as:

$$B_{p,\mathcal{C}}(x_p) := \frac{d_p}{\rho} + \min\left\{\frac{c_p}{\rho} \ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{d_p}{\rho} \ , 0\right\}\right\} (1-\tau) \max\left\{0, (1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right\}$$

While the financial institution is open $(x_t > x_p)$, its value function is the sum of the present value of cash-flows after taxes, the debt tax shield and the expected change in value when the closure option is exercised. That change is the product of the expected discount factor $(x_t/x_p)^{\beta}$ at the closure date and the difference between the bank's closure value $B_{p,\mathcal{C}}(x_p)$ and the combined present value of cashflows $(1-\tau)x_p/(\rho-\mu)$ and the tax rebate $\tau(c_p+d_p)/\rho$.

At closure $(x_t \geq x_p)$, the bank has value $B_{p,\mathcal{C}}(x_p)$ which comprises the closure value for depositors, non-deposit creditors and shareholders.

The change in the closure cost fraction from ϵ to ϵ_p affects the boundary $\overline{\xi}$ that separates strong from weak banks. Lemma 4 characterises the boundary with a PSA tool:

Lemma 4 (Bank's Type Boundary under PSA)**.** *Let:*

$$\overline{\xi}_p = \frac{\beta\epsilon_p - 1}{\beta - 1} \tag{61}$$

Then any bank whose capital regulation $\xi \geq \overline{\xi}_p$ is strong; any bank for which $\xi < \overline{\xi}_p$ is weak.

The intuition for Lemma 4 is similar to Lemma 1. The boundary $\overline{\xi}_p$ is the regulatory ratio that sets the impaired asset value under PSA equal to the present value of coupons at the bank's closure. Given Assumption 6, the bank's type boundary under PSA $\overline{\xi}_p$ is always lower than $\overline{\xi}$.

Figure 7 illustrates the change in boundaries along the range of the regulatory requirement ratio ξ. The introduction of the PSA increases efficiency at closure: it lowers the closure costs which consequently lowers the threshold that separates strong from weak banks. In this case, a subset of institutions that were previously deemed as weak banks are now considered strong. Hence, introducing the PSA tool makes some institutions better off.

Figure 8 illustrates the impact of PSA upon $\overline{\xi}$ for different growth rates μ of assets. For example, consider a bank that has a growth rate of assets $\mu = 0.01$ and faces a regulatory ratio $\xi = 0.25$. The decrease in closure costs from $\epsilon = 0.4$ to $\epsilon = 0.1$ moves the type boundary from $\overline{\xi} = 0.63$ to $\overline{\xi}_p = 0.16$ after the PSA. This weak bank becomes a strong institution with the introduction of the PSA tool.

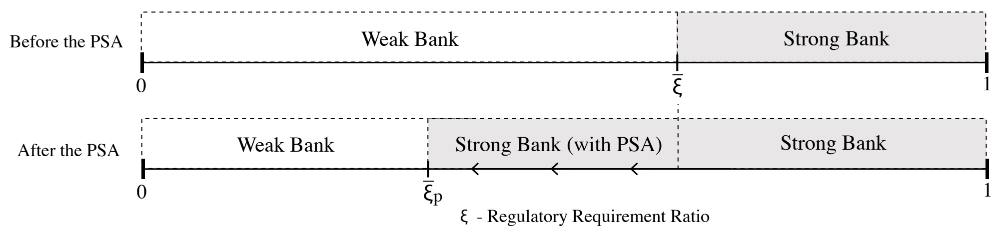


Figure 7: ***Impact of a change in the closure cost fraction from ϵ to ϵ_p.***

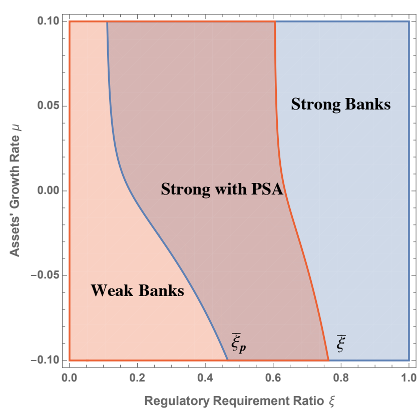


Figure 8: ***Example of impact of a change of closure cost fraction for different growth rates $\mu \in [-.1, .1]$.*** *The right region (blue area) comprises the banks that are strong before the PSA. The left region (red area) includes the weak banks that remain weak after the PSA. The intermediate region corresponds to banks that were weak when closures were privately managed but became strong after the PSA is introduced.* ***Fixed Parameters:*** *$\sigma = 0.05$; $\rho = 0.15$; $\epsilon = 0.6$; $\epsilon_p = 0.1$.*

5.2.2 Initial Capital Requirement

Under Assumption 4, the following Initial Capital Requirement (ICR) must hold:

$$c_p + d_p \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 \tag{62}$$

5.2.3 Non-Negative Coupons

By Assumption 1, each type of liability must be non-negative:

$$c_p \geq 0 \qquad \text{and} \qquad d_p \geq 0 \tag{63}$$

5.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_p, d_p) so as to maximise the total bank value $B_p(x_0)$ subject to the regulator's optimal closure trigger in condition (56), the ICR condition (62) and the non-negative coupons condition (63).

Optimisation Problem 3 (Bank)**.** *The optimal capital structure problem for the bank is:*

$$\begin{aligned}
&\left(c_p^*, d_p^*\right) && \in \underset{\{c_p, d_p\}}{\arg\max}\ B_p(x_0) && \\
&s.t. && x_p := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} && \textit{[Closure Trigger]} \\
& && c_p + d_p \leq \frac{\rho(1-\xi)}{\rho-\mu}x_0 && \textit{[Initial Capital Requirement]} \\
& && c_p \geq 0 \quad , \quad d_p \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}$$

5.3 The Private Sector Acquisition Equilibrium

Proposition 7 (Private Sector Acquisition Case Equilibrium)**.** *The equilibrium of the game with PSA is:*

$$\mathcal{E}_p = \left(c_p^*, d_p^*, x_p^*\right) \tag{64}$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}_p$$

Bank Optimal Decision. *The total periodic coupon $l_p^* = c_{di}^* + d_p^*$ is optimal iff:*

$$l_p^* = \min\left\{l_p^{**}, \bar{l}_p\right\} \tag{65}$$

where

$$l_p^{**} = \begin{cases} \frac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0, & \textit{for } \xi < \overline{\xi}_p \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)}x_0, & \textit{for } \xi \geq \overline{\xi}_p \end{cases} \tag{66}$$

and

$$\bar{l}_p = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{67}$$

Any combination of (c_p^*, d_p^*) is a feasible optimal solution, provided that it respects the following conditions:

$$d_p^* := l_p^* - c_p^* \qquad \textit{and} \qquad c_p^* = \begin{cases} 0 & , \quad \xi < \overline{\xi}_p \\ c_p^{**} & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{68}$$

with

$$c_p^{**} \in \begin{cases} \left(0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right) & , \quad for \quad l_p^{**} \geq \bar{l}_p \\ \\ \left(0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right) & , \quad for \quad l_p^{**} < \bar{l}_p \end{cases} \tag{69}$$

Regulator Optimal Decision. *At time T_p, cash-flows x_t hit the trigger x_p^*, the Regulator steps in and imposes a bankruptcy. That trigger is:*

$$x_p^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p^*+d_p^*}{\rho} \tag{70}$$

5.4 Discussion on the effects of introducing a PSA

I discuss the impact of introducing private sector acquisition tool in the equilibrium by comparing the PSA equilibrium in Proposition 7 against the costless deposit insurance equilibrium in Proposition 3. I write the equilibrium capital structure at time 0 for the DIS case as $CS_{di}^*(x_0) \equiv CS_{di}(x_0|\mathcal{E}_{di})$ and for the PSA case as $CS_p^*(x_0) \equiv CS_p(x_0|\mathcal{E}_p)$. I also write the equilibrium debt structure at time 0 for the DIS case as $DS_{di}^*(x_0) \equiv DS_{di}(x_0|\mathcal{E}_{di})$ and for the PSA case as $DS_p^*(x_0) \equiv DS_p(x_0|\mathcal{E}_p)$.

Proposition 8 (Change in the Equilibrium Total Coupon)**.** *Introducing the PSA tool weakly increases the equilibrium total coupon paid by the bank:*

$$l_p^* \geq l_{di}^*$$

Proposition 9 (Change in the Equilibrium Debt Structure)**.** *Introducing the PSA tool weakly increases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS_p^*(x_0) \geq DS_{di}^*(x_0) \tag{71}$$

Proposition 10 (Capital Structure Change)**.** *Introducing the PSA tool weakly decreases the equilibrium ratio between equity value and total bank value:*

$$CS_p^*(x_0) \leq CS_{di}^*(x_0) \tag{72}$$

Propositions 8 to 10 state that, in equilibrium, banks will not decrease leverage if the regulator introduces the private sector acquisition tool. These results are contingent on the type of the bank.

▹ **The Effect of PSA in Weak Banks.** If the bank is weak ($\xi < \bar{\xi}_p$), costless deposit insurance neutralises the impact of the PSA over the capital structure decision. This is because the bank only uses deposits in its liability structure. As discussed in Subsection 4.4, having deposits as an exclusive source of debt transfers the closure costs to the insurance fund. Therefore, introducing the PSA benefits the insurance fund by decreasing closure costs but it does not affect the bank's claimants. Hence, the weak bank does not change its capital structure:

$$CS_p^*(x_0|\xi < \bar{\xi}_p) = CS_{di}^*(x_0|\xi < \bar{\xi}_p)$$

▷ **The Effect of PSA in Strong Banks.** The decrease in closure costs from ϵ to ϵ_p incentivises the strong bank to increase the total coupon from l_{di} to l_p which leads to an increase in total debt. The bank does so to explore the debt tax shield.

However, an increase in debt has two important effects for shareholders: first, it decreases the residual present value of cash-flows available for shareholders $(1-\tau)(V(x_t) - L_p)$ and, second, it increases the optimal closure trigger $x_p > x_{di}$ (earlier closure). These two effects cause a decrease in the equity value.

Therefore, the strong bank lowers the equity over total bank value ratio:

$$CS_p^*(x|\xi > \overline{\xi}) \leq CS_{di}^*(x|\xi > \overline{\xi})$$

The PSA does not change the debt structure of the strong bank. Despite the increase in leverage, the mix between deposits and non-deposit liabilities should not change as a consequence of PSA. So, the debt structure in equilibrium should be:

$$DS_p^*(x_0|\xi > \overline{\xi}) = DS_{di}^*(x_0|\xi > \overline{\xi})$$

In the intermediate region ($\xi \in [\overline{\xi}_p, \overline{\xi}]$), where a bank is only strong under the presence of the PSA tool, the capital structure results are similar to the strong bank region. The bank increases leverage and suffers a decrease in the value of the equity claim; as a result, its equity/value ratio decreases:

$$CS_p^*(x_0|\xi \in [\overline{\xi}_p, \overline{\xi}]) \leq CS_{di}^*(x_0|\xi \in [\overline{\xi}_p, \overline{\xi}])$$

Banks in the intermediate region change their debt structure ratio so that they use both types of debt. The bank can always select a residual amount of non-deposit liabilities so that the debt structure does not materially differ significantly from the pre-PSA case. If a weak bank becomes strong with the PSA, it no longer has an incentive to use deposits as the exclusive source of debt.

From Propositions 8 to 10 and the above description, I get the following corollary:

Corollary 10.1. *Introducing the private sector acquisition tool incentivises all strong banks to increase leverage. Higher leverage decreases equity value and, hence, the capital structure ratio of strong banks decreases. Strong banks that were weak before the PSA no longer have an incentive to hold deposits as the only source of debt. The incentives of weak banks are not affected by the PSA.*

▷ **Implementing the change in the Capital Structure.** As in Subsection 4.4, the executive team must preserve the existing shareholders' value after the capital structure change. They can achieve this by buying back equity shares and paying with a claim over total liabilities.

If the bank wishes to maintain the debt structure, the claim over total liabilities must maintain the existing proportion of deposits and non-deposit liabilities. Using η_p to represent the share of ex-post liabilities held by existing shareholders, I write:

$$E_p(x_0|\mathcal{E}_p) + \eta_p\left(C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)\right) \geq E_{di}(x_0|\mathcal{E}_{di})$$

This expression requires the total value of the shareholders' claim after the change in capital structure, to be weakly greater than the equity value before the PSA is introduced.

Furthermore, shareholders should not receive more than the change in value caused by the new capital structure:

$$\eta_p\left(C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)\right) \leq B_p(x_0|\mathcal{E}_p) - B_{di}(x_0|\mathcal{E}_{di})$$

Using these two inequalities, I can find the admissible range of η_p that maintains existing shareholders satisfied with the capital structure change in response to the introduction of a PSA resolution tool.

$$\eta_p \in \left[\frac{E_{di}(x_0|\mathcal{E}_{di}) - E_p(x_0|\mathcal{E}_p)}{C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)}, \frac{B_p(x_0|\mathcal{E}_p) - B_{di}(x_0|\mathcal{E}_{di})}{C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)} \right] \tag{73}$$

In summary, the introduction of a private sector acquisition tools changes the incentives of all strong banks. This includes the banks that were strong before the PSA and the banks that became strong only after the PSA. The banks that remain weak are not affected by this resolution tool.

In terms of capital structure, the decrease in closure costs incentivises strong banks to leverage up the institution. However, this increase in leverage causes a decrease in the value of equity because higher debt coupons lower the shareholders' residual claim over cash-flows and increase the intervention trigger.

Regarding the debt structure, I see that banks that were strong before the PSA maintain the same debt structure. Banks that became strong only after the PSA no longer have an incentive to hold deposits as the sole source of debt. So, I expect these banks to add non-deposit liabilities though my model does not prescribe an optimal change. Finally, weak banks are unaffected by the PSA and do not change their debt structure.

In terms of implementation, the bank's executive team has to compensate existing shareholders if a change in the equilibrium capital structure decreases the value of the equity claim. Thence, it is expected them to be given a claim over the new debt that is at least as valuable as the equity claim value prior to the PSA.

6 The Bail-In Case

6.1 Elements of the Model

In this section, I introduce the Bail-in resolution tool, which I model as a debt to equity exchange imposed by the regulator.

▷ **The Bank.** The assets of the bank generate cash-flows represented by x_t following the GBM defined in equation (1) with drift $\mu \in (-\infty, \rho)$, volatility $\sigma > 0$ and discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (2).

The bank selects coupon c_{bi} for non-deposit creditors and a coupon d_{bi} for depositors at time 0. I write the total coupon as $l_{bi} = c_{bi} + d_{bi}$. The accounting value of total liabilities L_{bi} is the present value of total future coupon payments:

$$L_{bi} = \int_t^{+\infty} e^{-\rho(s-t)}(c_{bi} + d_{bi})ds = \frac{c_{bi} + d_{bi}}{\rho}, \tag{74}$$

The assets of the failing bank are bought by another institution at the closure time. As in Section 5.1, the impaired asset value after asset sales is $(1 - \epsilon_p)V(x_t)$, where ϵ_p is the closure cost fraction under the PSA tool. Furthermore, as in the previous sections, the corporate tax rate τ is levied on cashflows net of interest payments.

The bank's value function $B_{bi}(x_t)$ is the sum of the value of the claims:

$$B_{bi}(x_t) = E_{bi}(x_t) + C_{bi}(x_t) + D_{bi}$$

where $E_{bi}(x_t)$ is the equity claim value, $C_{bi}(x_t)$ is the non-deposit credit value and D_{di} is the value of deposits. When needed, I identify value functions according to the stage ti which they refer: I write $B_{bi,n}(x_t)$ for the bank's value function at stage $n \in \{1, 2, \mathcal{C}\}$. Stage 1 is the stage preceding the bail-in resolution; stage 2 succeeds the bail-in; and stage $\mathcal{C}$ is the closure stage. I use the same notation for value functions $E_{bi}(x_t)$, $C_{bi}(x_t)$ and D_{bi}.

The capital structure ratio $CS_{bi}(x_t)$ and the debt structure ratio $DS_{bi}(x_t)$ are defined as follows:

$$CS_{bi}(x_t) := \frac{E_{bi}(x_t)}{B_{bi}(x_t)} \qquad\qquad DS_{bi}(x) := \frac{C_{bi}(x_t)}{C_{bi}(x_t) + D_{bi}}$$

▷ **Bail-In.** The regulator imposes a Bail-in over the capital structure of the bank at the bail-in time T^1_{bi}, when cash-flows x_t first cross the trigger x^1_{bi}:

$$T^1_{bi} = \inf\left\{t \in [0, +\infty) : x_t \notin [x^1_{bi}, +\infty)\right\} \tag{75}$$

Following Proposition 1, the bail-in trigger x^1_{bi} is written as:

$$x^1_{bi} = \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_{bi} + d_{bi}}{\rho} \tag{76}$$

where $\xi \in (0, 1)$ is the regulatory requirement ratio presented in equation (4) and β corresponds to the elasticity of the bail-in option value to cash-flows depicted in equation (8).

I restrict the bail-in policy to a unique total conversion of non-deposit debt to new equity[25]. This implies that only non-deposit debt is considered as bail-inable, the entire claim is converted to

[25] My definition of bail-in can be relaxed to partial debt reduction with equity compensation. See Sarkar (2013) for a consideration on this extension.

equity, and that the bail-in conversion only occurs once. Consequently, the bank's debt comprises only deposits after the bail-in.

When bail-in occurs, the regulator issues new equity and converts non-deposit liabilities to new equity at a rate $\theta \in [0,1]$; old equity claims are converted to new equity at a rate $(1-\theta)$. That is, as in Sarkar (2013), the following identities hold at the time of the bail-in:

$$E_{bi,1}(x_{bi}) = (1-\theta)E_{bi,2}(x_{bi}) \tag{77}$$

and

$$C_{bi,1}(x_{bi}) = \theta E_{bi,2}(x_{bi}) \tag{78}$$

▹ **Bank's Closure.** The bank continues to operate after the bail-in until it reaches closure time T^2_{bi}, when the regulator imposes a PSA over the assets of the bank. Time T^2_{bi} is the time when cash-flows x_t first cross the trigger x^2_{bi}:

$$T^2_{bi} = \inf\left\{t \in [0,+\infty) : x_t \notin [x^2_{bi},+\infty)\right\} \tag{79}$$

Following Proposition 1 the closure trigger x^2_{bi} in the Bail-in case is:

$$x^2_{bi} = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{d_{bi}}{\rho} \tag{80}$$

▹ **Equilibrium.** Figure 9 depicts the three stages of the equilibrium. At $t=0$, the bank defines the capital structure by maximising the value function $B_{bi}(x_0)$ for a pair of coupons $(c_{bi}, d_{bi}) \in (0,+\infty)^2$. The bank operates from time 0 until time T^1_{bi}, when the regulator imposes a bail-in by converting non-deposit debt to equity. After the bail-in, the institution continues to operate until T^1_{bi}, when the regulator closes the bank, sells the assets through a PSA and pays depositors. The deposit insurance fund will cover the missing part of deposits if the asset sale is insufficient to cover this claim.

This is a sequential game with perfect information between these two players. I write equilibrium triggers ${x^1_{bi}}^* \equiv x^1_{bi}(c^*_{bi}, d^*_{bi})$ and ${x^2_{bi}}^* \equiv x^2_{bi}(d^*_{bi})$. The equilibrium tuple of the game is:

$$\mathcal{E}_{bi} := \left(c^*_{bi}, d^*_{bi}, {x^1_{bi}}^*, {x^2_{bi}}^*\right)$$

6.2 The Bank's Problem

6.2.1 Value Functions

The value of the bank satisfies the following accounting identity:

$$B_{bi}(x_t) = E_{bi}(x_t) + C_{bi}(x_t) + D_{bi} \quad \forall t \in [0, T^2_{bi})$$

Equity. For $t \in [0, T^1_{bi})$, the value of the equity claim is given by the following expression:

$$E_{bi}(x_t) := \mathbb{E}_t\left[\int_t^{T^1_{bi}} e^{-\rho(s-t)}(1-\tau)\left(x_s - c_{bi} - d_{bi}\right)ds\right] + \mathbb{E}_t\left[\int_{T^1_{bi}}^{T^2_{bi}} e^{-\rho(s-t)}(1-\tau)\left(x_s - d_{bi}\right)ds\right] + \\ + \mathbb{E}_t\left[\int_{T^2_{bi}}^{+\infty} e^{-\rho(s-t)} \max\left\{0, (1-\tau)\left((1-\epsilon_p)x^2_{bi} - d_{bi}\right)\right\}ds\right] \tag{81}$$

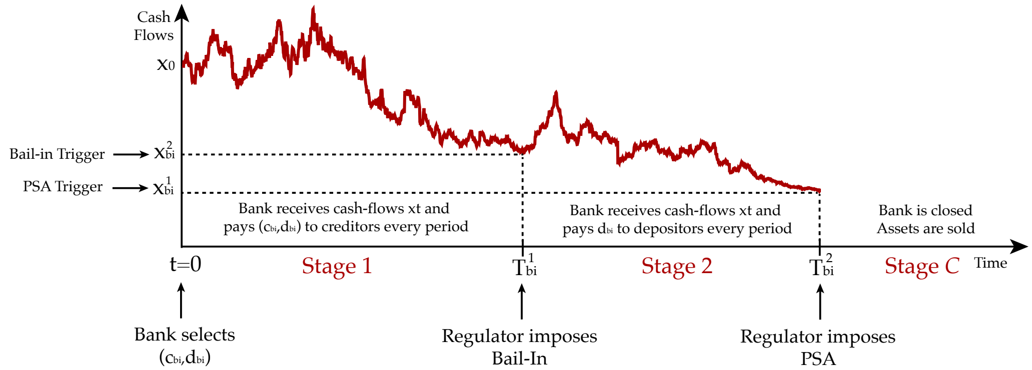


Figure 9: ***Timeline of the model for the two Scenarios.*** *Cash-flows evolve stochastically while gradually declining until reaching the bail-in trigger x^1_{bi} at time T^1_{bi}. The regulator converts non-deposit creditors to shareholders at the Bail-In trigger x^1_{bi} and prolong the firm's life until reaching the respective Private Sector Acquisition trigger x^2_{bi} at which bank is closed and the assets are sold.*

The three terms in Equation (81) are, respectively, the value that shareholders derive from the bank in stage 1 (before bail-in), stage 2 (after bail-in), and stage $\mathcal{C}$ (after closure by the regulator). Stage 1 runs from time t until bail-in occurs at time T^1_{bi}. During this stage, shareholders receive instantaneous cash-flows after interest payments and taxes equal to $(1-\tau)(x_t - c_{bi} - d_{bi})$. After bail-in occurs, the shareholders are augmented by the non-deposit creditors, who become shareholders as a result of the bail-in. Hence, because $c_{bi} = 0$ after bail-in, shareholders receive cash flow $(1-\tau)(x_t - d_{bi})$. Shareholders are protected by limited liability. Hence, during stage $\mathcal{C}$, which runs from closure at time T^2_{bi} to infinity, shareholders receive the larger of zero and the after-tax present value of the impaired cash flow less the deposit payment: that is, the larger of 0 and the present value of $(1-\tau)((1-\epsilon_p)x^2_{bi} - d_{di})$.

Using standard techniques developed in Dixit and Pindyck (1994), the equity value function $E_{bi}(x_t)$ can be written as follows:

$$E_{bi}(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right) + (1-\tau)\left(\frac{x_t}{x^1_{bi}}\right)^{\beta}\left[\frac{c_{bi}}{\rho} - \theta\left(\frac{x^1_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right)\right] - \\ \quad -(1-\tau)(1-\theta)\left(\frac{x_t}{x^2_{bi}}\right)^{\beta}\min\left\{\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}, \epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right\} & \text{for } x_t > x^1_{bi} \\ (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) - (1-\tau)\left(\frac{x_t}{x^2_{bi}}\right)^{\beta}\min\left\{\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}, \epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right\} & \text{for } x^1_{bi} \geq x_t > x^2_{bi} \\ (1-\tau)\max\left\{0, (1-\epsilon_p)\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right\} & \text{for } x_t \leq x^2_{bi} \end{cases} \tag{82}$$

Like Equation (81), value function (82) has three cases that correspond to the three stages of this version of the model. While the bank is operational and has not yet been subjected to a bail-in ($x_t > x^1_{bi}$), the value of the claim of initial shareholders equals the equity value. In this case, the equity value is the sum of three terms: the first term, $(1-\tau)(x_t/(\rho-\mu) - (c_{di}+d_{di})/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to all creditors; the second term is the expected change in value if the regulator imposes a bail-in to the bank; the third term is the expected change in value if the regulator closes to the bank.

The expected change in equity value caused by the bail-in is the product of the expected discount factor $(x_t/x^1_{bi})^\beta$ and the change in equity value if the bank suffers the bail-in. This change in value is the difference between the present value of future non-deposit coupons $(1-\tau)c_{bi}/\rho$ and the share of residual cash-flows to be given to new shareholders $(1-\tau)\theta\left(x^1_{bi}/(\rho-\mu)-d_{bi}/\rho\right)$. This means that, at the bail-in, old shareholders no longer owe future coupon payments to non-deposit credit claimers but they lose a share θ of residual cash-flows to equity that is given to these credit claimers.

The expected change in equity value for old shareholders if the regulator closes the bank is the product of the expected discount factor $(x_t/x^2_{bi})^\beta$ and a share $(1-\theta)$ on the change in equity value at closure. In detail, the change in equity value for old shareholders is the difference between the closure equity value $(1-\tau)(1-\theta)\max\left\{0,(1-\epsilon_p)x^2_{bi}/(\rho-\mu)-d_{bi}/\rho\right\}$ and the after tax present value of cash-flows net of debt payments $(1-\tau)(1-\theta)\left(x^2_{bi}/(\rho-\mu)-d_{bi}/\rho\right)$.

After the bail-in and before the regulator closes the bank ($x^1_{bi}\geq x_t>x^2_{bi}$), equity is owned by both old and new shareholders. Its value is divided in two terms: the first term, $(1-\tau)(x_t/(\rho-\mu)-d_{di}/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to depositors; The second term is the expected change in value if the regulator closes to the bank.

When the bank is closed ($x_t\leq x^2_{bi}$), shareholders receive the closure payoff. Because shareholders have limited liability, this payoff is at least zero. They receive a zero payoff when the impaired asset value from the PSA $(1-\epsilon_p)V(x^2_{bi})$ is insufficient to cover the present value of deposit coupons d_{bi}/ρ. If the impaired asset value covers the present value of coupons, all shareholders obtain the residual value after paying depositors and taxes.

Non-deposit Liabilities. The value function for the non-deposit credit claim is:

$$C_{bi}(x_t):=\mathbb{E}_t\left[\int_t^{T^1_{bi}}e^{-\rho(s-t)}c_{bi}ds\right] \tag{83}$$

Non-deposit creditors earn a periodic coupon c_{bi} until the bank is bailed-in at time T^1_{bi}. The non-deposit claim ceases to exist upon bail-in, when non-deposit creditors are converted to new equity owners.

$$C_{bi}(x_t)=\begin{cases}\frac{c_{bi}}{\rho}+\left(\frac{x_t}{x^1_{bi}}\right)^\beta\left[\theta(1-\tau)\left(\frac{x^1_{bi}}{\rho-\mu}-\frac{d_{bi}}{\rho}\right)-\frac{c_{bi}}{\rho}\right]- & \\ \qquad -\theta(1-\tau)\left(\frac{x_t}{x^2_{bi}}\right)^\beta\min\left\{\frac{x^2_{bi}}{\rho-\mu}-\frac{d_{bi}}{\rho},\epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right\} & \text{for } x_t>x^1_{bi}\\ 0 & \text{for } x_t\leq x^1_{bi}\end{cases} \tag{84}$$

Before the bail-in is implemented ($x_t>x^1_{bi}$) the value of non-deposit credit comprises three terms: the first term, c_{bi}/ρ, is the present value of coupons. The second term is the expected change in the value for non-deposit creditors if the regulator imposes a bail-in. This is the product between the expected discount factor $(x_t/x^1_{bi})^\beta$ and the change in creditors' value if a bail-in occurs. The change in value if a bail-in occurs is the difference between the share of cash-flows net of deposits and taxation $\theta(1-\tau)(x_{bi}/(\rho-\mu)-d/\rho)$ received by non-deposit creditors and the present value of coupons c_{bi}/ρ they forfeit.

The third term in the case where $x_t<x^1_{bi}$ is the expected change in the non-deposit creditors value if the bank closes. This is the product between the expected discount factor $(x_t/x^2_{bi})^\beta$ and the change in the creditors' value at closure. Creditors become the new shareholders after T^1_{bi} and have a share θ of the equity value. The creditors' value at closure is the difference between the

closure equity value $\theta(1-\tau)\max\left\{0,(1-\epsilon_p)x_{bi}^2/(\rho-\mu)-d_{bi}/\rho\right\}$ and the after tax present value of cash-flows net of debt payments $\theta(1-\tau)\left(x_{bi}^2/(\rho-\mu)-d_{bi}/\rho\right)$.

After the bail-in time T_{bi}^1, the financial institution does not hold non-deposit liabilities and this claim ceases to exist.

Deposits. Deposits have a constant value since they are protected by the deposit insurance scheme and because bail-ins are not extended to this class of debt. That value is:

$$D_{bi}=\frac{d_{bi}}{\rho}, \qquad \text{for } t\geq 0 \tag{85}$$

Bank Value. I obtain the following value function for the financial institution by summing the equity, the non-deposit credit, and the deposits claims:

$$B_{bi}(x_t)=\begin{cases}(1-\tau)\frac{x_t}{\rho-\mu}+\tau\frac{c_{bi}+d_{bi}}{\rho}-\tau\left(\frac{x_t}{x_{bi}^1}\right)^{\beta}\frac{c_{bi}}{\rho}-(1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^{\beta}\min\left\{\frac{x_{bi}^2}{\rho-\mu}-\frac{d_{bi}}{\rho},\epsilon_p\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_t>x_{bi}^1\\ (1-\tau)\frac{x_t}{\rho-\mu}+\tau\frac{d_{bi}}{\rho}-(1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^{\beta}\min\left\{\frac{x_{bi}^2}{\rho-\mu}-\frac{d_{bi}}{\rho},\alpha\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_{bi}^1\geq x_t>x_{bi}^2\\ (1-\tau)\max\left\{0,(1-\epsilon_p)\frac{x_{bi}^2}{\rho-\mu}-\frac{d_{bi}}{\rho}\right\}+\frac{d_{bi}}{\rho} & \text{for } x_t\leq x_{bi}^2\end{cases} \tag{86}$$

While the financial institution is open ($x_t>x_{bi}^1$) its value function is the sum of the present value of cash-flows after taxes, the tax shield of debt, the expected change in total value when the bail-in option is exercised, and the expected change in value when the closure option is exercised.

The expected value change associated to the bail-in option is the product between the expected discount factor $(x_t/x_{bi}^1)^{\beta}$ and the loss of the tax shield of non-deposit liabilities $\tau c_{bi}/\rho$ at the bail-in time T_{bi}. The expected value change associated to the bank's closure is the product between the expected discount factor $(x_t/x_{bi}^2)^{\beta}$ and the closure value.

After the bail-in ($x_{bi}^1\geq x_t>x_{bi}^2$), the bank generates after tax cash-flows and a deposit tax shield $\tau d_{bi}/\rho$. The last term in this expression is expected change in value given closure.

The institution is subjected to a PSA at closure ($x_t\leq x_{bi}^2$) which will yield a total value $(1-\tau)(1-\epsilon_p)V(x_{bi}^2)$, provided all depositors are fully covered by the impaired asset value. If the impaired asset value is lower than deposits, the bank value is equal to the deposits value D_{bi}, since these claimants are covered by the deposit insurance scheme.

In the Bail-in case, a strong bank is a bank whose impaired asset value is always sufficient to cover the deposits obligations given the PSA closure costs. From Lemma 4, the boundary that separates strong banks from weak banks is:

$$\overline{\xi}_p=\frac{\beta\epsilon_p-1}{\beta-1}$$

where the institution is considered strong if $\xi\geq\overline{\xi}_p$ and weak if $\xi<\overline{\xi}_p$. This boundary stays unchanged from Section 5.2.

Consider the following assumption:

Assumption 7 (No Worse-Off Principle)**.** *No shareholder or creditor shall incur greater losses under a bail-in than they would have incurred if the bank had been subjected to normal bankruptcy proceedings.*

Assumption 7 accounts for the No Worse-Off (NWO) principle as defined in Article 74 of Directive 2014/59/EU[26]. The Bail-In conversion share θ must be such that each class of bail-inable investors can only accept a Bail-In if they receive a share of value that is weakly better than suffering a private-sided bankruptcy.

Lemma 5. *The No Worse-Off Principle is satisfied when*

$$\theta \in \left[\frac{\min\left\{(1-\epsilon)\frac{x_{bi}^1}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\}}{E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)}, 1 - \frac{\max\left\{(1-\tau)\left[(1-\epsilon)\frac{x_{bi}^1}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\}}{E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)} \right] \cap [0,1] \qquad (87)$$

Lemma 5 states that the bail-in conversion share θ must respect the participation constraints of equity-owners and non-deposit creditors at the bail-in trigger time. That is, it must respect these claimants' alternative of facing a bank's closure at time T_{bi}^1 while suffering a closure cost fraction ϵ.

▷ **Bail-in Conversion Share.** Now consider the partition of equity value between old and new shareholders when the bank suffers the bail-in.

The existing literature, such as Sarkar (2013) and Pawlina (2010) relies upon a Nash bargaining game to determine the optimal share of value. This literature assigns exogenous bargaining power to every party and determines the way that the value is divided. However, this approach does not work in my model because, to be consistent with reality, the conversion rule must be defined by financial authorities that are completely independent of the failing banks' stakeholders.

There are several alternatives to determine the conversion share θ under a bail-in intervention[27]. I adopt $\overline{\theta}$ as the conversion share rule for my model:

Assumption 8. *The bail-in conversion share is:*

$$\overline{\theta} \equiv \frac{c_0}{\rho\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right)} \qquad (88)$$

Assumption 8 states that the regulator selects a bail-in conversion share $\overline{\theta}$ that is equal to the ratio between the present value of future coupon payments to non-deposit creditors and the value of the bank excluding deposits. Conversion share must respect the NWO principle in Lemma 5 and, so, the proposed share $\overline{\theta}$ must always be limited by the boundaries presented in condition (87).

This rule is consistent with the financial regulator's goal of guaranteeing that shareholders are the first to suffer the consequences of a bail-in. With the rule in Equation (88), the value of

[26] See Bank Recovery and Resolution Directive.

[27] For example, following Moulin (2004), one can use an egalitarian rule. In this case, I would select a share θ that solves the equality:

$$(1-\theta)\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right) - \max\left\{(1-\tau)\left[(1-\epsilon_p)\frac{x_{bi}^1}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\} = \theta\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right) - \min\left\{(1-\epsilon_p)\frac{x_{bi}^1}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\}$$

This condition sets the share of value of non-deposit creditors equal to the share of value of shareholders excluding their respective value had they entered a bankruptcy. Hence, this alternative considers the shareholders' and non-deposit creditors' opportunity cost of the bail-in, i.e. the bankruptcy alternative.

non-deposit liabilities is unaffected ex-ante by the bail-in rule: they receive $\overline{\theta} E_{bi,2}(x_{bi}^1)$, which is equal to c_{bi}/ρ. For the period $t \in [0, T_{bi}^1)$, the expected loss of value caused by bail-in is experienced by shareholders, while the value of non-deposit liabilities depends only upon the expected present value of coupons. Hence, conversion rule $\overline{\theta}$ secures that non-deposit creditors never suffer a decrease in the expected value of the claim until time T_{bi}^1 nor capture any rents beyond the promised present value of the coupons until T_{bi}^1. When non-deposit creditors become shareholders they may absorb losses if cash-flows continue to erode ($x_t < x_{bi}^1$).

6.2.2 Initial Capital Requirement

Under Assumption 4, the Initial Capital Requirement (ICR) must hold:

$$c_{bi} + d_{bi} \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 \tag{89}$$

6.2.3 Non-Negative Coupons

By Assumption 1, each type of liability must be non-negative:

$$c_{bi} \geq 0 \qquad \text{and} \qquad d_{bi} \geq 0 \tag{90}$$

6.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_{bi}, d_{bi}) so as to maximise the total bank value $B_{bi}(x_0)$ subject to the regulator's optimal bail-in trigger in condition (76), his optimal closure trigger in condition (80), the ICR condition (89) and the non-negative coupons condition (90).

Optimisation Problem 4 (Bank)**.** *The optimal capital structure problem for the bank is:*

$$\begin{aligned}
(c_{bi}^*, d_{bi}^*) \quad & \in \underset{\{c_{bi}, d_{bi}\}}{\arg\max}\; B_{bi}(x_0) & \\
s.t. \qquad & x_{bi}^1 := \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_p + d_p}{\rho} & \textit{[Bail-In Trigger]} \\
& x_{bi}^2 := \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_p + d_p}{\rho} & \textit{[Closure Trigger]} \\
& c_{bi} + d_{bi} \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 & \textit{[Initial Capital Requirement]} \\
& c_{bi} \geq 0 \quad , \quad d_{bi} \geq 0 & \textit{[Non-Negative Coupons]}
\end{aligned}$$

6.3 The Bail-in Equilibrium

Proposition 11 (Bail-in Case Equilibrium)**.** *The equilibrium of the game with Bail-in is:*

$$\mathcal{E}_{bi} := \left(c_{bi}^*, d_{bi}^*, {x_{bi}^1}^*, {x_{bi}^2}^* \right)$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}_p$$

Bank Optimal Decision. *The total periodic coupon $l_{bi}^* = c_{bi}^* + d_{bi}^*$ is optimal iff:*

$$l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}_{bi}\} \tag{91}$$

where the interior solution is $l_{bi}^{**} = c_{bi}^{**} + d_{bi}^{**}$ *with*

$$d_{bi}^{**} = \begin{cases} (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{92}$$

and

$$c_{bi}^{**} = \begin{cases} (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{93}$$

Otherwise, the bank selects $\overline{l}_{bi} = \overline{c}_{bi} + \overline{d}_{bi}$ *where*

$$\overline{d}_{bi} = \begin{cases} \frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & \text{for } \xi < \overline{\xi}_p \\ \frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{94}$$

and

$$\overline{c}_{bi} = \begin{cases} \frac{\rho}{\rho-\mu}(1-\xi)\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}\right)x_0 & \text{for } \xi < \overline{\xi}_p \\ \frac{\rho}{\rho-\mu}(1-\xi)\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{95}$$

Regulator Optimal Decision. *At time* T_{bi}^1*, cash-flows* x_t *hit the trigger* ${x_{bi}^1}^*$*, the Regulator steps in and imposes a Bail-in. That trigger is:*

$${x_{bi}^1}^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_{bi}^* + d_{bi}^*}{\rho} \tag{96}$$

At time T_{bi}^2*, cash-flows* x_t *hit the trigger* ${x_{bi}^2}^*$*, the Regulator steps in and imposes a Private Sector Acquisition. That trigger is:*

$${x_{bi}^2}^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{d_{bi}^*}{\rho} \tag{97}$$

6.4 Discussion on the effects of introducing Bail-Ins

I write the equilibrium capital structure at time 0 for the PSA case as $CS_p^*(x_0) \equiv CS_p(x_0|\mathcal{E}_p)$ and for the Bail-In case as $CS_{bi}^*(x_0) \equiv CS_{bi}(x_0|\mathcal{E}_{bi})$. I also write the equilibrium debt structure at time 0 for the PSA case as $DS_p^*(x_0) \equiv DS_p(x_0|\mathcal{E}_p)$ and for the Bail-In case as $DS_{bi}^*(x_0) \equiv DS_{bi}(x_0|\mathcal{E}_{bi})$.

Proposition 12 (Change in the Equilibrium Total Coupon)**.** *Introducing the Bail-in tool weakly increases the equilibrium total coupon paid by the bank:*

$$l_{bi}^* \geq l_p^*$$

Proposition 13 (Change in the Equilibrium Debt Structure)**.** *Introducing the Bail-in tool weakly increases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS^*_{bi}(x_0|\xi < \overline{\xi}_p) \geq DS^*_p(x_0|\xi < \overline{\xi}_p) \tag{98}$$

The change in debt structure is undefined for strong banks.

Proposition 14 (Capital Structure Change)**.** *Introducing the Bail-in tool weakly decreases the equilibrium ratio between equity value and total bank value:*

$$CS^*_{bi}(x_0) \leq CS^*_p(x_0) \tag{99}$$

Propositions 12 to 14 state that, in equilibrium, the introduction of a bail-in tool does not cause banks to decrease leverage. The precise effect depends upon whether the bank is weak or strong.

▹ **The Effect of Bail-Ins in Weak Banks.** Introducing bail-ins affects the equilibrium debt structure and capital structure of weak banks (that is, banks for which $\xi < \overline{\xi}_p$). Without bail-ins, weak banks only have deposit debt. With them, they may also choose to assume some non-deposit liabilities. Hence, the weak banks' debt structure weakly increases with the introduction of the bail-in:

$$DS^*_{bi}(x_0|\xi < \overline{\xi}_p) \geq DS^*_p(x_0|\xi < \overline{\xi}_p)$$

The bail-in tool can be valuable for weak banks because it creates a *recapitalisation option*. The reason is that, if the bank holds non-deposit liabilities, the regulator has an option to change the capital structure by converting non-deposit credit to equity. Doing so prolongs the life of the institution and gives the bank a second chance to recover the cash-flows and possibly to prevent a future closure.

In the PSA case, the weak bank was restricted to select the optimal leverage by balancing the expected costs of closure against the benefit of the tax shield over deposits. The bail-in tool incentivises the weak bank to increase the total coupon from l^*_p to l^*_{bi} so as to use the recapitalisation option to increase the bank's debt capacity and, hence, the tax shield it derives from borrowing.

Because it increases debt capacity and, hence, the tax shield of debt, the bail-in causes a weak decrease in the bank's equilibrium capital/value ratio:

$$CS^*_{bi}(x_0|\xi < \overline{\xi}_p) \leq CS^*_p(x_0|\xi < \overline{\xi}_p)$$

▹ **The Effect of Bail-Ins in Strong Banks.** In the PSA case, the institution is free to define any mix of positive deposits and non-deposit coupons provided that the identity $l^*_p = c^*_p + d^*_p$ is respected. In contrast, when the bail-in tool is introduced, the strong bank must select a unique bail-in equilibrium coupon mix (c^*_{bi}, d^*_{bi}). My model therefore yields no predictions regarding the change in the debt structure.

Similarly to the weak bank's case, the introduction of the bail-in tool increases the total leverage of the strong institution causing an increase in value and a decrease in the equity value due to the lower after-taxes residual cash-flow $(1-\tau)(V(x_t) - L_{bi})$. Hence, I have:

$$CS^*_{bi}(x_0|\xi \geq \overline{\xi}_p) \leq CS^*_p(x_0|\xi \geq \overline{\xi}_p)$$

From Propositions 12 to 14 and the above description, I get the following corollary:

Corollary 14.1. *Introducing the bail-in tool incentivises all banks to increase leverage. Higher leverage decreases equity value and increases value and, hence, the capital structure ratio of banks decreases. Weak banks will not decrease the debt structure ratio. Strong banks can increase, decrease or maintain their debt structure.*

7 Quantitative Results

In this section, I compare the bail-in case against the base case. I calibrate the model, provide plots for the equilibrium coupons, equilibrium capital structure and equilibrium debt structure (see Appendix).All claims are evaluated at time $t = 0$ in equilibrium. That is, under the equilibrium tuple $\mathcal{E}$ in the base case or the equilibrium tuple $\mathcal{E}_{bi}$ in the bail-in case.

▷ **Capital and Debt Structure Ratios.** For the base case, the equilibrium capital structure is $CS^*(x_0)$ and the equilibrium debt structure is $DS^*(x_0)$. Explicitly:

$$CS^*(x_0) := \frac{E(x_0|\mathcal{E})}{B(x_0|\mathcal{E})} \qquad DS^*(x_0) := \frac{C(x_0|\mathcal{E}_i)}{D(\mathcal{E}_i) + C(x_0|\mathcal{E})}$$

As in the previous section, I write the equilibrium Capital Structure for the Bail-In case as $CS^*_{bi}(x_0)$ and the equilibrium Debt Structure as $DS^*_{bi}(x_0)$:

$$CS^*_{bi}(x_0) := \frac{E_{bi}(x_0|\mathcal{E}_{bi})}{B_{bi}(x_0|\mathcal{E}_{bi})} \qquad DS^*_{bi}(x_0) := \frac{C_{bi}(x_0|\mathcal{E}_{bi})}{D_{bi}(\mathcal{E}_i) + C_{bi}(x_0|\mathcal{E}_{bi})}$$

To understand the role of the options of regulatory intervention (closure and bail-in) over the capital and debt structures of the bank, I write the accounting versions of the capital structure ratio and the debt structure ratio using the accounting value of the claims. To recall, the accounting value of a claim is the present value of all future receivables to be earned by the owner of the claim. For example, in the base case, the accounting value of equity is the after tax present value of cash-flows net of interest payments $(1-\tau)\,(V(x_0) - L)$, the accounting value of deposits is D and the accounting value of non-deposit liabilities is $L - D$.

Hence, for the base case, I write the equilibrium Accounting Capital Structure ratio as $ACS^*(x_0)$ and the Accounting Debt Structure ratio as $ADS^*(x_0)$, where:

$$ACS^*(x_0) := \frac{1-\tau)\,(V(x_0|\mathcal{E}) - L(\mathcal{E}))}{(1-\tau)\,(V(x_0|\mathcal{E}) - L(\mathcal{E})) + L(\mathcal{E})} = \frac{(\tau-1)(\mu(l^*) - \rho(l^* - x_0))}{\tau(\mu-\rho)l^* + \rho(\tau-1)x_0}$$
$$ADS^*(x_0) := \frac{L(\mathcal{E}) - D(\mathcal{E})}{L(\mathcal{E})} = \frac{c^*}{l^*}$$

For the bail-in case, the equilibrium Accounting Capital Structure ratio is $ACS^*_{bi}(x_0)$ and the Accounting Debt Structure ratio is $ADS^*_{bi}(x_0)$, where:

$$ACS^*_{bi}(x_0) := \frac{1-\tau)\,(V_{bi}(x_0|\mathcal{E}_{bi}) - L_{bi}(\mathcal{E}_{bi}))}{(1-\tau)\,(V_{bi}(x_0|\mathcal{E}_{bi}) - L_{bi}(\mathcal{E}_{bi})) + L_{bi}(\mathcal{E}_{bi})} = \frac{(\tau-1)(\mu(l^*_{bi}) - \rho(l^*_{bi} - x_0))}{\tau(\mu-\rho)l^*_{bi} + \rho(\tau-1)x_0}$$
$$ADS^*_{bi}(x_0) := \frac{L_{bi}(\mathcal{E}_{bi}) - D_{bi}(\mathcal{E}_{bi})}{L_{bi}(\mathcal{E}_{bi})} = \frac{c^*_{bi}}{l^*_{bi}}$$

▷ **Calibration Values.** Table 1 summarises the values used for the model calibration. I study two different setups for the bank's assets. I define the "good state" bank as the bank that has assets with a positive growth rate of cash-flows $\mu = 0.01$ and low uncertainty $\sigma = 0.01$. Differently, the bank in the "bad state" has assets that generate cash-flows with negative growth rate $\mu = -0.01$ and high uncertainty $\sigma = 0.05$. My choice of a growth rate μ close to 0 is consistent with the recent estimations of Detragiache et al. (2018). They identify that the assets' profitability of European banks in the periods are very close to 0 and often bellow 1%, during and after the crisis. My choice for the uncertainty σ is also in line with Chen et al. (2017) who estimate the standard deviation from the model calibration for 17 financial institutions to range from 0.038 to 0.086.

Consistent with previous literature (Hugonnier and Morellec 2017, de Mooij and Gaetan 2015), I set the corporate tax rate to $\tau = 0.25$. This is in line with Langedijk et al. (2014) who estimate that corporate income tax rates among European banks range from 0.10 to 0.40.

The closure costs fraction $\epsilon = 0.20$ for the base case (private-sided closure) is close to the estimates of James (1991) for financial firms and to Andrade and Kaplan (1998) and Korteweg (2010) for non-financial firms. In summary, these papers estimate that bankruptcy costs range from 10% to 30% of firm value. Furthermore, I define a PSA closure cost $\epsilon_p = 0.15$ which is cautiously close to ϵ due to the lack of research on banks' asset sales intermediated by resolution authorities.

The values of the other parameters are as follows. The regulatory requirement ratio is $\xi = 0.15$ and it is inspired by the EBA guidelines on capital and liquidity adequacy requirements for early intervention triggers[28]. The exogenous discount rate is $\rho = 0.15$ which is slightly higher than, for example, Hugonnier and Morellec (2017). I choose a higher rate to ensure the comparative statics presented in Figures 10 to 17 do not have indeterminate results across the ranges studied. Moreover, lower discount rates do not change my results significantly. Finally, I arbitrarily set the initial cash-flow level to $x_0 = 50$.

Parameter	Common	Good State	Bad State	Description
μ	-	0.01	-0.01	Expected instantaneous growth of cash-flows
σ	-	0.01	0.05	Instantaneous volatility
τ	0.25	-	-	Tax rate
ϵ	0.20	-	-	Closure cost fraction in the base case
ϵ_p	0.15	-	-	Closure cost fraction under the PSA
ξ	0.15	-	-	Regulatory requirement ratio
ρ	0.15	-	-	Exogenous discount rate
x_0	50	-	-	Initial cash-flow level

Table 1: *Calibration parameters.*

7.1 Equilibrium Coupons

▷ **Base Case - Coupon.** Consider Figure 10. The equilibrium total coupon l^* increases with the cash-flows growth rate μ and decreases with the regulatory requirement ratio ξ. For the calibration values described in Table 1, changes in the cash-flows' uncertainty σ and changes in the corporate tax rate τ do not seem to affect the equilibrium total coupon significantly.

The equilibrium total coupon level l^* always presents an interior solution when the bank is in the bad state. That is because the negative cash-flow growth and the higher uncertainty increase the costs of closure which forces the bank to choose a lower equilibrium total coupon when maximise the bank's value.

Moreover, the bad state has a higher minimum levels of non-deposit coupons than the good state (orange dashed lines). As described in Subsection 3.3, banks with lower growth rate of assets and higher standard deviation σ must hold a higher proportion of non-deposit liabilities in its coupon structure in order to prevent a bank-run.

▷ **Bail-in Case - Coupon.** Consider Figure 13. Similarly to the base case, calibration shows that

[28]See Art. 27 of 2014/59/EU and in EBA/GL/2015/03. In summary, these guidelines suggest that national regulators can select regulatory capital ratios by imposing a buffer over the capital adequacy ratio imposed to banks.

the equilibrium total coupon l^*_{bi} increases with the cash-flows growth rate μ and decreases with the regulatory requirement ratio ξ, while changes in uncertainty σ and in the tax rate τ do not seem to affect the significantly the equilibrium total coupon.

In the bail-in case, the good state bank is almost exclusively financed by deposits while the bad state bank tends to include non-deposit liabilities in their coupon structure for positive growth rates. Non-deposit liabilities are included in the coupon structure of the bad state bank because they can be used to induce the regulator to recapitalise the institution through the Bail-in tool and gain time for the cash-flows to recover. This is particularly relevant for institutions with high volatility of cash-flows σ and cash-flows' growth rates μ that are not extremely negative. A bank with these characteristics has a high likelihood of recovering from a near closure situation and, so, it has an incentive to hold non-deposit liabilities that can be converted to equity at the bail-in time.

Subfigures 13.e and 13.f show that changes in the tax rate do not seem to affect the equilibrium coupon structure with the exception of the bad state for very low tax rates. In this case, the lower tax rates causes a decrease in the bail-in costs for the bank $-\tau c_{bi}/\rho(x_0/x^1_{bi})^\beta$ and, so, it is cheaper for the bank to induce a recapitalisation via the bail-in resolution tool.

The equilibrium total coupon l^*_{bi} decreases if the regulatory requirements ratio ξ increases (subfigures 13.g and 13.h). In this case, higher capital requirements increase the bail-in and closure triggers. This forces the bank to select a lower equilibrium total coupon to avoid earlier interventions. The bank in the bad state includes non-deposit liabilities if the regulatory requirement ratio is high, that is, if $\xi > 15.4\%$. A higher ξ ratio increases the sensitivity of the closure trigger to changes in deposits $\partial^2 x^1_{bi}/\partial d_{bi}\partial\xi > 0$. Therefore, having one additional unit of deposit coupon d_{bi} becomes very expensive if the regulatory requirement ratio is high. This prompts the bad state bank to reduce the deposit coupon and increase the non-deposit coupon c_{bi} as ξ increases.

7.2 Equilibrium Capital Structure

▷ **Base Case - Capital Structure.** Consider Figure 11. The capital structure ratio varies negatively with growth μ and tax rate τ and it varies positively with uncertainty σ and capital requirement ratio ξ.

As shown in subfigures 11.a and 11.b, lower growth rate μ prompts the firm to decrease leverage and to increase equity causing the capital structure ratio to increase. This is particularly the case for negative values of μ. Furthermore, the capital structure ratio $CS^*(x_0)$ is lower than the accounting capital structure $ACS^*(x_0)$ for growth rates that are not extremely negative. However, this reverts as μ takes extremely negative value. In this case, the closure option becomes positively valued for equity-holders because, for extremely negative growth rates, the limited liability property limits the losses of shareholders at closure.

The positive relationship between the equity/value ratio and cash-flow uncertainty σ is represented in Subfigures 11.c and 11.d. In the good state, the capital structure ratio $CS^*(x_0)$ is always lower than the accounting capital structure $ACS^*(x_0)$ while, in the bad state, high σ values inverts the relationship inverts. This is because the closure option becomes positively valued for equity-holders.

Furthermore, subfigures 11.e and 11.f show that higher taxation causes a decrease in the ratio of equity over value in both states. Both equity and total value functions decrease as the tax rate increase. However, the total value function decreases at a slower pace due to the tax shield of debt.

Subfigures 11.g and 11.h reveal that higher capital requirements increase the capital structure ratio. Higher capital requirements ratio ξ increase the intervention trigger x_b. To prolong the bank's lifetime the bank decreases the equilibrium total coupon. As a result, the weight of equity in the capital structure increases.

▷ **Bail-In Case - Capital Structure.** Consider Figure 14. In the bail-in case to growth (μ) and uncertainty (σ) do not affect the capital structure significantly. Overall, the ratio of equity over value remains stable around 10.5%-12.5%.

As presented in subfigure 14.a and 14.b, the capital structure ratio $CS^*_{bi}(x_0)$ stays bellow the accounting capital structure $ACS^*_{bi}(x_0)$ for growth rates $\mu > -0.026$ in the good state and $\mu > -0.018$ in the bad state. For lower values of μ, the limited liability property of equity limits the losses of shareholders at closure which makes the closure option positively valued for shareholders and, hence, it leads to $CS^*_{bi}(x_0) > ACS^*_{bi}(x_0)$.

The gap between the capital structure ratio and the accounting capital structure ratio widens significantly for $\mu \in [-0.026, 0]$ in the good state and for $\mu \in [-0.018, 0.05]$ in the bad state. This is a consequence of a surge of non-deposit liabilities in the equilibrium coupon structure, observable in subfigures 13.a and 13.b. All else constant, a higher level of equilibrium non-deposit coupon increases the conversion rate $\overline{\theta}$ in condition (87) which translates into a lower share of new equity available to initial shareholders.

Subfigures 14.c and 14.d show that the bank's equity over value ratio does not change significantly given changes in uncertainty. The difference between the capital structure ratio $CS^*_{bi}(x_0)$ and the accounting capital structure $ACS^*_{bi}(x_0)$ is essentially caused by the presence of non-deposit liabilities in the equilibrium coupon structure which lowers the conversion share of new equity offered to existing shareholders at the bail-in time.

Subfigures 14.e and 14.f show the negative effect of taxes on capital structure. In fact, higher tax rates decrease equity and total bank value. However, equity decreases faster than total value with higher tax rates. The slower decrease of bank's total value function is a consequence of the tax shield of debt. Higher tax rates increase tax shields in the bank's total value, alleviating the negative impact of taxation in the after-tax cash-flows.

Finally, subfigures 14.g and 14.h show that the equilibrium capital structure increases with the regulatory requirement ratio ξ. This is a consequence of the negative relationship between equilibrium total coupon l^*_{bi} and the regulatory requirement ratio ξ observed in subfigures 13.g and 13.h. Higher requirement ratios ξ increases the closure trigger, forcing the bank to decrease the equilibrium coupon. As a result, the residual cash-flow is higher available to shareholders increases causing an increase in the equity value function $E_{bi}(x_0|\mathcal{E}_{bi})$ and an increase in the capital structure ratio $CS^*_{bi}(x_0)$ and the accounting capital structure $ACS^*_{bi}(x_0)$.

▷ **Base Case vs Bail-In Case - Capital Structure Comparison.** Figure 16 combines Figures 11 and 14 to show the aggregate effect of transiting from the base case (before the regulatory changes) to the bail-in case (after the regulatory changes). Consistent with Propositions 6, 10 and 14, the main result of Figure 16 is:

Result 1. *The introduction of the Costless Deposit Insurance, Private Sector Acquisition tool and the Bail-In tool causes a decrease in the ratio of Equity over Total Bank Value in equilibrium. This decrease is greater if the bank is in a bad state than in a good state.*

In equilibrium, the financial institution selects higher total coupons after the regulatory changes. Higher coupons increases the debt value and decreases the weight of Equity in the Total Bank Value. As discussed in the previous sections, each one of these tools contributes to this effect. First, the introduction of costless deposit insurance makes the "No Bank-Run" condition obsolete for weak banks. Any closure costs that would be supported by depositors are transferred to the deposit insurance fund. Strong banks are not affected by this first regulatory change. Second, the decrease of closure costs fraction ϵ to ϵ_p under the Private Sector Acquisition tool decreases the termination costs for strong banks, prompting these institutions to increase leverage. Third, the bail-in offers a recapitalisation option that, when exercised, prolongs the life of the bank. Bail-Ins can affect both types of banks but not all banks will choose to hold non-deposit liabilities. Only those banks that benefit from a recapitalisation option will choose to include non-deposit liabilities. This is translated in a weak increase in the total liabilities of the bank.

Figure 16 demonstrate the combined effects of these regulatory changes. Subfigures 16.a to 16.d show that the capital structure decrease from the base case to the bail-in case is smaller if the growth rate of cash-flows μ is greater and the cash-flow uncertainty σ is lower. Furthermore, observing 16.e and 16.f, the gap between the capital structures of the base case and the bail-in case diminishes as tax rates τ and regulatory ratios ξ increase.

In summary, anks in a bad state show a sensitivity of their equilibrium capital structure to changes in regulation that is higher than the banks in a good state banks. A major factor for this difference is the expected discount factor of closure in the base case $(x_0/x_b^*)^\beta$ which increases significantly with lower growth rate μ and higher volatility σ.

7.3 Equilibrium Debt Structure

▷ **Base Case - Debt Structure.** My model does not provide a unique debt structure ratio in the base case. Figure 12 shows the minimum debt structure ratios admissible in the base case while Figure 17 shows the region of feasible equilibrium debt structures. The debt structure has an upper-bound of 100% non-deposit liabilities and a lower-bound debt structure defined by the lowest amount of non-deposit liabilities necessary to avoid a depositors-run, as defined in Lemma 2.

The debt structure region in the bad state is always contained in the good state debt structure region for all studied parameters. The No-Bank Run condition, discussed in Subsection 3.3.2, forces the bank in the bad state to hold a higher level of minimum non-deposit coupons. The bad state bank has less freedom when choosing its debt structure.

▷ **Bail-In Case - Debt Structure.** As displayed in Figure 15, the debt structure ratio $DS_{bi}^*(x_0)$ is always equal to the accounting debt structure $ADS_{bi}^*(x_0)$ This is a consequence of the chosen bail-in conversion rate $\overline{\theta}$ defined in condition (88). A higher conversion rate $(\theta > \overline{\theta})$ would lead to a debt structure ratio higher than the accounting debt structure. In this hypothetical case, non-deposit claimants would capture a higher share of new equity at the bail-in intervention. This would increase their expected total value but not their present value of cash-flows. The opposite case, $\theta < \overline{\theta}$, holds with opposite results.

Debt structure follows the same change pattern as the coupon structure. As seen in subfigures 15.a) and 15.b, the debt structure ratio is highly sensitive to changes in the growth rate μ in critical subregions. These regions are coincident with a higher share of value $\overline{\theta}$. For instance, in both subfigures, small differences of 2 percentage points around the growth rate of 0% can lead to changes in the debt structure of more than 10p.p. The bank chooses to include non-deposit liabilities as the growth rate moves improves. For every additional unit of deposits, the expected value of the closure option becomes comparatively more expensive for the bank than the expected value of the bail-In option and, hence, the bank can increase value by including non-deposit liabilities in the debt structure.

As seen in subfigures 15.c and 15.d, the reaction of debt structure to changes in the uncertainty parameter σ depends on the state of the bank. Higher uncertainty in the good state, increases the debt structure ratio. Differently, higher uncertainty in a bad state regime decreases the debt structure ratio. The specific combination of high uncertainty and high growth induces the highest levels of non-deposit liabilities in the coupon structure which causes a high debt structure.

The debt structure ratios $DS^*_{bi}(x_0)$ and $ADS^*_{bi}(x_0)$ vary negatively with tax rate τ. This ratio is less sensible to shocks in the good state than in the bad state. Observing subfigures 15.e and 15.e, we see that a tax rate increase of 10p.p. causes a reduction of less than 0.5 p.p. in the debt structure ratio for a bank in the good state and of, approximately, 7p.p. in the bad state

The debt structure ratio varies positively with the requirement ratio ξ but the state of the bank is crucial for the sensitivity of $DS^*_{bi}(x_0)$ to ξ. Subfigure 15.f shows that a shock of 10p.p. in the capital requirement ratio yields a maximum increase of 1p.p.. This sensitivity is significantly higher in the bad state where, as shown in subfigure 15.g, an increase in the regulatory requirement ratio can result, at most, in an increase of nearly 20p.p., depending on the region under study.

▷ **Base Case vs Bail-In Case - Debt Structure Comparison.** Figure 17 combines Figures 12 and 15 to show the aggregate effect of transiting from the base case (before the regulatory changes) to the bail-in case (after the regulatory changes). Consistent with Propositions 5, 9 and 13, the main result of Figure 17 is:

Result 2. *For low values of regulatory requirement ratio ξ, the introduction of regulatory changes causes a decrease in the ratio of equilibrium Non-Deposit Liabilities over Total Liabilities. The effect is undefined for high levels of ξ.*

In the base case, the range is bounded below by the No Bank-Run condition and above by the Initial Capital Requirement condition. The bail-in case has a unique solution for the debt structure ratio $DS^*_{bi}(x_0)$. Inspecting subfigures 17.a to 17.f, one sees that the equilibrium debt structure in the bail-in case is always lower than the minimum bound for the equilibrium Debt Structure in the base case. This is a consequence of the chosen regulatory requirement ratio $\xi = 0.15$.

However, for higher levels of ξ, approximately $\xi \geq .18$, I cannot corroborate this effect (subfigures 17.g and 17.h) because the debt structure ratio in the bail-in case is contained within the boundaries of the base case debt structure ratio. In the base case, a higher regulatory requirement ratio ξ expands the range of admissible debt structures by making the no-bank run condition obsolete while, in the bail-in case, a higher regulatory requirement ratio ξ increases the relative costs of deposits over non-deposit liabilities in the debt structure. That is because, the sensitivity of the closure option cost to changes in deposits increases with ξ. So, in order to increase increase debt, the bank prefers to use non-deposit liabilities instead of deposits because the firsts do not affect the closure of the institution.

8 Concluding Remarks

This paper studies the impact of three key regulatory changes over the capital structure of financial institutions: a strong increase in the deposit limits in Deposit Insurance Schemes; the introduction of Private Sector Acquisitions, that is, the sale of the bank's assets by the regulator; and the Bail-In tool which converts non-deposit creditors to equity.

I build a capital structure model in continuous time in which a financial institution trades-off debt tax shields and bankruptcy costs in the shadow of possible regulatory intervention. The regulator decides when to close the bank and, if the bail-in tool is available, it also decides when to impose a bail-in. I use this framework to examine the relationship between the three regulatory changes and the ratio of equity over total value as well as the ratio of non-deposit liabilities over total liabilities. . Both of these ratios changed after the financial crisis.

My model yields several predictions. First, my analysis suggests that each of the above regulatory changes leads to an increase in bank leverage. Costless deposit insurance incentivises the use of deposits as a single source of debt so that any potential closure costs are met by the insurance fund; the private sector acquisition tool reduces bank closure costs and so incentivises the bank to assume more debt; finally, the bail-in tool postpones the bank's closure pushing and so causes the bank to borrow more.

Second, my results suggest that the increase in the equity/value ratio observed in Europe after 2010 was not caused by the above regulatory changes, because all induce the bank to borrow more as a proportion of its asset value. The observed increase in banks' equity/value ratios can therefore be explained by the higher capital requirements introduced by Basel III. Moreover, it is likely that the actual sensitivity of the equity/value ratio to changes in capital requirements may be higher than apparent given the aforementioned effect of the three regulatory changes under study.

Third, my results indicate that the decrease in the ratio of non-deposit liabilities to total liabilities can be explained by the changes in the regulatory environment, namely, by the strengthening of deposit insurance. My analysis suggests that, for a realistic parameter calibration, deposits become the bank's preferable financing source.

Despite the preference for deposits, my results suggest that deposit insurance does always crowd-out non-deposit liabilities from the debt structure of banks. That is because bail-ins generate a valuable *recapitalisation option* when the bank's capital structure is hard-to, or impossible, to change. The institution can induce a future recapitalisation by adding non-deposit liabilities to the debt structure that will be converted to equity when the regulator imposes the bail-in.

I suggest two paths of future research. The first is to study the other dimensions of the bail-in tool. In my model, I restrict this resolution tool to a debt-to-equity exchange. However, bail-ins comprise other actions, for example, managerial turnover. It would be interesting to study the incentives of a manager to define the capital structure of his bank knowing that, in the future, if the bank is bailed-in he can be fired and incur in reputation costs. I conjecture that this would reduce the incentives of this agent to include non-deposit liabilities in the debt structure.

A second research path relates with the systemic implications of Global Systemically Important Banks (GSIFIs). Each GSIFI bank is subject to specific capital requirements aiming at curbing the increased systemic risk. Hence, it would be interesting to study the GSIFI case by modelling the expected spillover effect of a GSIFIs performance over the financial system and make endogenous the bank-specific regulatory requirement ratio.

A Appendix - Overview of the Regulatory Changes

In this paper I consider three major regulatory changes: Bail-in tool, Liquidation tool and the increased limit for Deposit Insurance. I now detail and characterise these new elements.

▹ **Bail-In.** Bail-ins empower the regulator (or resolution authority) with the legal right of changing the property and control rights across different (bailinable) claimants, e.g. unsecured senior liabilities, subordinated debt and equity, without the need for court authorisation (Chennells and Wingfield 2015). In practice, the authority can erase the claims of shareholders and unprotected creditors or even change the nature of these claims, e.g. debt-to-equity swap. Sometimes, bail-ins comprise as well a set of restructuring measures (e.g. change in management, or sale of parts of the business) to improve the bank's viability, which is a unique trait unavailable to contingent convertibles.

The embryonic case of a bail-in occurred in Cyprus after its sovereign crisis of 2012 where, given the levels of bank capital necessities and the public debt refinancing needs, the most plausible solution to avoid a banking collapse was a bail-in of unprotected depositors[29] (Michaelides 2014).

▹ **Private Sector Acquisition.** A Private Sector Acquisition (European Central Bank) or Private Sector Purchase (Bank of England)[30]. The resolution authority uses this tool to apply an orderly sale of the assets, using its proceedings to cover the liabilities of the bank.

This tool is distinct from the pure bankruptcy because the resolution proceedings are managed by the resolution authority and not by an assembly of creditors. In theory, this control leads to lower costs of fire sales as the authority can avoid negative externalities associated with resolutions managed by creditors, i.e. "rush to the courthouse" and "holdout" externalities (Roubini 2004). The authority has sufficient control to search the best opportunity to sell the assets hence generating higher sales proceedings for the creditors.

▹ **Increase in the limit for Deposit Insurance.** Deposit insurance is a mechanism by which a guarantor (usually an institution funded by the government or by its associated banks) can cover the losses of depositors in the event of a bankruptcy of some financial institution. If the insured limit is sufficiently high, such that most depositors are covered, this mechanism can avert bank-runs given that depositors are not at risk of supporting bankruptcy costs in their role of creditors (Iyer and Puri 2012). Deposit Insurance Schemes in developed economies are not particularly new. However, they were subject to significant increases in the deposit insurance limit during and after the financial crisis. For some European countries, the regulatory change[31] Because these increments in the limit for deposit insurance were in most countries non-trivial, I treat this aspect in my model in an extreme version, i.e. the bank under study goes from no deposit insurance to full deposit insurance.

[29] In detail, for a total write-down of 5.8 billion Euros, the best option involved imposing a stability levy over residents and non-residents' depositors. In spite of the existing opposition and the lack of a clearly defined legal framework within the EU, this bail-in to banks was carried out for the first time for the first time in Eurozone. A levy was determined at 6.75% for deposits up to 100 thousand euros, and 9.9% above that level (Zenios 2015). For a critical discussion of bail-ins as a resolution mechanism, check Avgouleas and Goodhart (2014).

[30] In detail, the BoE defines a Private Sector Purchase as a partial or total alienation of the assets of a financial institution to a private sector buyer, whereas the ECB defines Private Sector Acquisition as the transfer of parts of the business (assets, shares, liabilities) to one or more purchasers, regardless of shareholders? approval (Art. 38 and 39 of BRRD and Art. 24 of SRM). Note that the ECB considers the possibility of transferring the liabilities to the private acquirer. I choose to ignore this later feature in my model and restrict to the asset sale.

[31] EU member states were obliged to increase their deposit insurance level to a minimum of EUR50,000 in 2009, and then to an homogenised level of EUR100,000 in 2011.

Table 1: Summary of Resolution Frameworks

Area[a]	USA	United Kindgom	European Union
Interveners	(1) The chartering authority of a financial institution, i.e. a state banking department or the US Office of the Comptroller of the Currency, decides when to put a firm under resolution. (2) The Federal Deposits Insurance Corporation (FDIC) receives the firm under legally independent functioning from its deposit insurance role.	After consultation of the HM Treasury, the supervisor (Prudential Regulation Authority) and the resolution authority (Bank of England) decide on the resolution. For investment firms the prudential supervisor is usually the Financial Conduct Authority.	Each member state has an administrative organism holding full resolution powers, e.g., Finanzmarktaufsicht in Austria or Banca d'Italia in Italy. The Single Resolution Board is an independent european authority which can enforce resolution for banks within the jurisdictions of the the Single Resolution Mechanism.
Resolution Trigger	Banks are required to be adequately capitalized with risk-based capital ratios of 8% or higher[b] which triggers a warning it is is breached. If the ratio lowers to 6%, the regulator steps in and impose corrective actions. The bank is considered to be critically undercapitalized at 2%, for which the regulator will close it and transfer the assets to the FDIC.	Resolution can occur before a firm is balance sheet insolvent. It occurs when the financial institution: (1) is failing or on the verge of failing, for instance, when it cannot meet the minimum requirements in terms of quality capital and liquidity levels; (2) is reasonably unlikely to recover with actions taken outside the resolution regime.	Occur when: (1) a bank is failing; and/or (2) no other measure taken by the stakeholders (e.g. voluntary debt-to-equity swap) is likely to ensure the institution's viability in a timely matter or without causing instability in the financial system[c]. Some events can trigger early resolution[d], namely, operational risk events (e.g. rogue trading), deterioration in the level of eligible liabilities, strong outflow of funds (e.g. deposits) due to reputation damage, rating downgrades.
Goals of Resolution[e]	The decision of the receiver should aim at maximizing the return on the assets of the bank under resolution, and to minimize the costs of the insurance fund that may arise in the case of a liquidation, while preserving the interests of protected creditors.	To safeguard banking services while maintaining financial stability, public confidence, and avert using public funding. If an orderly resolution cannot exist, the BoE aims to mitigate the effects of failure, e.g. fire-sales costs, preserve credit operations and securing access to deposits.	Respond in an efficient manner to a bank under financial distress, ensure financial stability and mitigate the costs for society, i.e. public resources supported by taxpayers.
Available Tools	**(1) Purchase and Assumption (P&A):** FDIC sells assets to an Assuming Institution, e.g. a healthy bank. **(a) Whole Bank P&As:** The FDIC asks for bids on all assets of the failing bank on a "take is as it is" basis, without guarantees. **(b) P&As with Optional Shared Loss:** The FDIC, as a receiver, shares partial assets' losses with the AI. **(c) Bridge Bank P&As:** The failed institution is transferred to a temporary structure controled by the FDIC and chartered by the OCC. **(2) Deposit Payoff (DP)** occurs when the FDIC acts as a deposit insurer, and thus pays these claimants up to the insured maximum amount (see next table). **(a) Straight DP:** FDIC determines the amount of insured deposits and pays it directly. Used when liquidation is the least costly resolution. **(b) Insured Dep. Transfer:** FDIC transfers secured depositors to a local bank which directly pays the claims. **(c) Deposit Insurance National Bank:** It is a temporary bank created to ensure that depositors can access and manage their funds.	**(1) Stabilization Tools** to promote financial stability and the continuation of critical business functions: **(a) Private Sector Purchase** consists on a partial or total alienation of the assets of a financial institution to a private sector buyer; **(b) Bridge Bank** is used to transfer part/total of the business to a subsidiary of the BoE. This is an intermediary step with the goal of future sale. **(c) Bail-In** use the claims of the failing institution as a resource to recapitalize and absorb its losses. **(2) Complementary Tools** can only be used in conjunction with the previous three stabilization tools: **(a) Asset Separation** is used to transfer assets and credits to a separated asset management vehicle. **(b) Bank Administration Procedure** When part of the business is not immediately transferred to a private buyer or into a bridge bank, the "residual bank" is put into administration, so that basic services are continued. **(3) Insolvency** If the public interest is not met, the an insolvency expert is appointed to manage the wind-down of the assets of the bank.	**(1) Private Sector Acquisition** (Art. 38 and 39 of BRRD and Art. 24 of SRM), consisting on the transfer of parts of the business (assets, shares, liabilities) to one or more purchasers, regardless of shareholders' approval; **(2) Bridge Bank** (Art. 40 and 41 of BRRD, and Art. 25 of SRM) is a controlled temporary structure, used by the resolution entity to encompass parts of the business; **(3) Asset Separation** (Art. 42 of BRRD and Art 26 of SRM) of assets that can affect market stability to an asset management vehicle[f] controlled by the resolution authority.; **(4) Bail-In** (Art. 43 to 55 of BRRD) Recapitalization of the financial institution through mandatory debt-to-equity swap; **(4) Government Stabilization** (Art 56 to 58 of BRRD) Temporary (and last resort) public support.

[a] The resolution framework of the USA is consigned in the Dodd-Frank Act - Title II (2009), of the UK in the Banking Act 2009 (amended by the BRRD), and in the European Union by the Bank Recovery and Resolution Directive (BRRD) and the Single Resolution Mechanism Regulation (SRMR) in 2014.

[b] For additional details on risk-based capital ratios see: https://www.fdic.gov/regulations/safety/manual/section2-1.pdf

[c] For details on when a bank is considered as failing see: https://www.eba.europa.eu/documents/10180/1085517/EBA-GL-2015-07+GL+on+failing+or+likely+to+fail.pdf

[d] See https://www.eba.europa.eu/regulation-and-policy/recovery-and-resolution/guidelines-on-early-intervention-triggers

[e] The FSB provides a set of common key attributes and goals for an effective resolution of financial institutions at http://www.fsb.org/2011/11/r_111104cc/

[f] During the sovereign crisis, AMVs often implied that losses were absorbed by taxpayers via government support (**?**).

Table 2: Summary of Bail-In Tools and Depositors Protection Schemes

Area	USA	United Kindgom[a]	European Union[b]
Hierarchy of Creditors	The allocation of proceedings (or responsibilities, in the inverse order) should obey the following hierarchy: (1) Secured Depositors and Preferred Creditors; (2) Depositor Class on a pro-rata basis (this includes uninsured depositors and FDIC in the event of proceedings allocation); (3) General Creditors; (4) Subordinated Creditors; (5) Shareholders.	The insolvency hierarchy is: (1) Fixed charge holders, i.e. mortgage securities, covered bonds, collateralised positions; (2) Fees and expenses held by liquidators and by resolution authority; (3) Ordinary Preferential Creditors, i.e. protected depositors; (4) Secondary Preferential Creditors, i.e. uninsured depositors; (5) Floating charge holders that do not fit in the first category; (5) Unsecured Senior Creditors; (6) Unsecured Subordinated creditors; (7) Interest post-insolvency; (8) Shareholders.	Member states have discretionary powers to govern the ranking in insolvency hierarchy provided that, as established in Art. 108 of BRRD, the following general order is followed: (1) Covered deposits and "deposit guarantee schemes subrogating to the rights and obligations of covered depositors in insolvency"; (2) Deposits from "natural persons and MSM-sized firms" which exceeds the coverage level; (3) Ordinary unsecured, non-preferred Creditors.
Bail-In Specifics	The Dodd-Frank Act - Title II defines new approaches to banking resolution, allowing for an orderly resolution through instruments that may, in some specific cases, replicate the outcome of a bail-in. However, this legislation does not define or characterize a bail-in tool in the 'sensu stricto' created by the BRRD tool.	The bail-in tool defined in BRRD was included in the UK legislation under the Financial Services Act in 2013 (Schedule 2) and thus most of the rules are, for now, similar to the European case. **Solutions:** Open Bank and Closed Bank Resolutions, though the first is more likely to be used in the UK[c]. **Exceptions:** Certain financial arrangements may be excluded from a bail-in, e.g. collateral, or . **Requirements (MREL):** Calibration of MREL will be done on a firm-specific basis. On the first phase, financial Institutions will comply with BoE's Interim Requirements[d] by 2020, with different timings and requirements depending on their category (G-SIB, D-SIB, or others), e.g. from Jan-2019, UK G-SIBs must hold at least 16% of RWAs or 6% of leverage exposures. On the second phase, the BoE will impose tougher requirements by 2022. **Minimum Bail-In:** The write-off of liabilities myst reach a threshold of 8%.	A bail-in is applied, without needing court approval, by (1) writing-off equity and (2) writing-off or imposing a debt-to-equity swap to liabilities that are not ex-ante expressly excluded (e.g. covered deposits, employee remuneration). **Solutions:** (1) An Open Bank Resolution where a failing institution is recapitalized via the write-down of liabilities or a debt-to-equity swap ensuring its continuation; (2) A Closed Bank Resolution where toxic assets stay in the existing institution (bad bank) and a new entity (good bank) is created to encompass clean assets. Credit claims with no systemic impact can either stay in the old bank or move into the new one, being swapped to equity or written-down. **Exceptions:** Critical creditors may be excluded from a bail-in when it is impossible to consider them without creating market instability or affecting the continuity of core operations. **Requirements (MREL):** Banks should, at all times, hold a minimum level of own funds and bail-in eligible liabilities. Such level is determined by each member-state resolution authority[e]. **Minimum Bail-In:** To avoid instability, the BRRD permits ending the burden upon reaching a threshold of 8% of total liabilities (plus capital). After securing the minimum bail-in, resolution funds may assume 5% of the losses.
Depositors Protection	The FDIC acts as an insurance on amounts up to 250.000 USD per depositor on each ownership category, e.g., Retirement accounts, Employee Benefit Plan accounts, per FDIC-secured financial institution.	The Financial Services Compensation Scheme (FSCS) provides insurance coverage up to 85.000 GBP per deposit claims (personal or firm) against financial institutions under resolution from 30th of January of 2017[f].	The Directive 2014/49/EU[g] legislates that each member state is responsible for ensuring, within its territory, the existence of at least one Deposit Guarantee Scheme, with a coverage amount of 100.000 Euros per depositor.

[a] See http://www.prarulebook.co.uk/ and the BoE's guide on resolution at www.bankofengland.co.uk/financialstability/Documents/resolution/apr231014.pdf

[b] This table is based on the Directive 2014/59/EU available at http://data.europa.eu/eli/dir/2014/59/oj

[c] As conjectured by (?)

[d] As of November 2016. For full details go to http://www.bankofengland.co.uk/publications/Pages/news/2016/082.aspx

[e] The general criteria used to determine the minimum level of bail-in eligible securities is defined at the Art.45 (6) of BRRD.

[f] Recently updated at https://www.fscs.org.uk/what-we-cover/compensation-limits/

[g] See http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:32014L0049

Table 3: Examples of Bail-Ins (and other resolution tools)

Example 1:	**The Cypriot Depositors Bail-in (2012 onwards)**
Institutions	Bank of Cyprus (BoC) and Cyprus Popular Bank (Laiki).
Authorities	The Central Bank of Cyprus (CBC) acted initially as the supervisor and later on as the resolution authority, after the Cyprus Resolution Law. The Troika (ECB, IMF, and EC) also intervened in the process.
Tools	Sale of Business from Laiki to BoC and Bail-In of BoC, complemented with a Bailout from Troika.
Concerns	How to implement unavoidable restructuring measures using untested tools (bail-in) without a carefully carved legal ground to support them. This was ”patient zero” for bail-ins of financial institutions.
Summary	By the beginning of the crisis, BoC and Laiki's were highly exposed to greek sovereign bonds. In 2011, the Greek Private Sector Involvement originated a write-off of around 4 Billion Euros on their balance sheets. Cyprus delayed the financial assistance request up until June of 2012, after increasing economic imbalances, and after almost 1 year of failure to access capital markets. The agreement with Troika, conditional on stress tests to banks, was only accomplished by March 2013 and involved a bailout of EUR 10 Billion without support to BoC and Laiki (which needed approximately EUR 7 Billion). This delay in negotiations allowed capital (deposits) outflows of around 10 to 17 Billion Euros. Subsequently, and after failing to legally approve a tax of 9.9% for uninsured depositors and of 6.75% for insured depositors, the parliament passed the Cyprus Resolution Law on March 22, endowing the CBC with resolution powers. Laiki was hastily resolved with write-offs of shareholders, bondholders and uninsured depositors. Critical assets, insured depositors and 9 Billion Euros from the Emergency Liquidity Assistance were transferred to BoC. All greek related assets and liabilities, e.g. loans and deposits, were sold at discount to Piraeus Bank at around 3.2 Billion Euros. Approximately 37.5% of BoC's uninsured deposits were then converted to equity. Temporary capital controls were further imposed to avoid additional deposits outflows.
Example 2:	**Spanish Cajas Bail-In (2009 onwards)**
Institutions	Ten Savings banks: Caixa Bank, BBVA, Ibercaja, Banco Sabadell, Unicaja, Mare Nostrum, BFA, NCG Banco, Novacaixa Galicia, Liberbank, and BFA Bankia.
Authorities	The Fund for Orderly Bank Restructuring (FROB) managed the restructuring processes, and the Deposit Guarantee Fund (DGF) acts as a deposit insurer, though it had a further role in this process.
Tools	Public Funding, Bail-Ins and Asset Separation via an asset management company (SAREB).
Concerns	Include the BRRD in legislation and carefully implement the resolution. Minimize the public financial burden.
Summary	Cajas are savings banks where the figure of shareholder is replaced by the ”mutualist” associate, without dividends, and governed by private and public stakeholders. Cajas needed public support and thus, from 2009 to 2012, the FROB provided financial aid of approximately EUR 15 Billion to eight of the listed banks, combined with EUR 7.9 Billion from the DGF. In mid 2012, the EU facilitated a total of EUR 41.3 Billion to recapitalize the 10 banks through the FROB and to inject capital in SAREB. By February 2013, approximately 94% of that amount had been used. All banks were obliged to transfer critical assets, at discount, to SAREB, e.g. illiquid or non-performing real state loans. The operation totaled EUR 50.8 Billion exchanged against sovereign guaranteed SAREB bonds. Shareholders were wiped out, preference shares and perpetual subordinated creditors were converted to common equity. Other subordinated creditors could choose between equity or senior debt. All subordinated claimants received an haircut. The bail-in totaled EUR 13.6 Billion. To reduce litigation with private claimants, the DGF set up a liquidation mechanism, offering to buy at discount the equity instruments from converted claimants.
Example 3:	**Danish Bail-In (2015 onwards)**
Institutions	Andelskassen Bank.
Authorities	The Danish Financial Supervisory Authority (FSA) acts as regulator, and the Finansiel Stabilitet Company (FSC) holds the functions of receiver and resolution managing entity.
Tools	Open Bank Resolution combining a Bail-In with Bridge Bank.
Concerns	Secure an orderly and smooth resolution by granting uninterrupted critical services and full access to deposits. Minimize the litigation costs of the uninsured claims affected by the bail-in.
Summary	From 2008 to 2012, Denmark introduced 5 bank packages focused on securing financial stability through structured resolution framework for these institutions. In 2015, the BRRD was legally embodied. FSA asked Andelskassen to submit a recovery plan by March 2015. In October 5, 2015 the FSA concluded that a private restructuring was no longer feasible. To prevent failure and given that resolution was in the public interest, the regulator intervened immediately. In that same day, FSC took over, changed the board and the management team with immediate effect, and created a bridge bank, called Broinstitut I A/S, fully detained by the resolution authority. Resolution was initiated on the afternoon of that day and thus critical banking services stayed uninterrupted. The FSA proceeded with a write-off of all equity, all subordinated credit claims and part of the senior credit claims. While some uninsured depositors had to support a write-off, the Danish Deposit Guarantee Scheme protected insured depositors up to 100.000 Euros. In succession, the FSA used the Resolution Fund to inject DKK 37.5 million of new capital and ensure solvency. No claimant affected by the write-off proceeded with litigation against the resolution procedure.

B Appendix - Regulator Option Value

I derive the regulator's closure option $O(x_t)$ presented in section 3.2. Recall the regulator's utility function defined in (5):

$$R(x_t) = (1-\xi)V(x_t) - L \tag{100}$$

Following Dixit and Pindyck (1994), I write the Bellman equation of the closure option as:

$$\rho O(x_t)dt = \mathbb{E}_t[dO]$$

Expanding the RHS using Ito's Lemma and rearranging, I obtain the following differential equation that must be satisfied by $O(x_t)$:

$$\frac{1}{2}\sigma^2 x^2 O''(x_t) + \mu x O'(x_t) - \rho O(x_t) = 0 \tag{101}$$

In addition $O(x_t)$ must satisfy the following conditions:

$$\lim_{x_t \to +\infty} O(x_t) = 0 \tag{102}$$

$$O(x_b) = -R(x_b) \tag{103}$$

Condition (102) comes from the observation that the regulator would never close an over-performing bank. The closure option becomes worthless if the bank's cash-flows are exceedingly high. Condition (103) is the value matching condition which states that, at x_b, the regulator exercises the closure option and stops facing (dis)utility $R(x_b)$.

I propose the following solution for this ODE:

$$O(x) = Ax^{\beta} + Bx^{\beta+} \tag{104}$$

where A, B, $\beta < 0$ and $\beta_+ > 1$ are constants. I replace this solution proposal into ODE (101). Using conditions (102) and (103), I arrive to the following results:

$$A = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{1}{x_b}\right)^{\beta} \tag{105}$$

$$B = 0 \tag{106}$$

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \tag{107}$$

In detail, $B = 0$ is imposed since condition (102) requires the closure option to converge to zero when cash-flows go to infinity. I combine this with the value matching condition (103) to obtain the solution for constant A. Finally, β is the negative root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$ that arises when I replace the proposed solution $O(x)$ in the ODE. β_+ is the positive root which is inconsequential since $B = 0$. Replacing these solutions in (104), I get the closure option's value:

$$O(x_t) = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{x_t}{x_b}\right)^{\beta}$$

C Appendix of Base Case

Proof of Proposition 1. Recall from function (7), the value of the closure option:

$$O(x_t) = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{x_t}{x_b}\right)^{\beta}$$

My problem is:

$$x_b \in \arg\max_{x_b} O(x_t)$$

Obtaining the FOC over $O(x_t)$ and rearranging, I obtain:

$$x_b = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c+d}{\rho}$$

□

Proof of Lemma 1. Consider the present value of all future payments to all creditors (non-deposit creditors and depositors):

$$\mathbb{E}_{T_b}\left[\int_{T_b}^{+\infty} e^{-\rho(s-T_b)}\,(c+d)\,dt\right] = \frac{c+d}{\rho}$$

From Proposition (1), the optimal intervention trigger selected by the Regulator is:

$$x_b(\bar{\xi}_1) := \frac{\beta(c+d)(\rho-\mu)}{(\beta-1)(1-\bar{\xi})\rho}$$

Set the value of the assets' proceedings under bankruptcy at the trigger x_b equal to the total creditors' present value of payments, i.e. solve $(1-\epsilon)\frac{x_b(\bar{\xi})}{\rho-\mu} = \frac{c+d}{\rho}$ for $\bar{\xi}$. □

Proof of Lemma 2. Consider the present value of all future payments to depositors:

$$\mathbb{E}_t\left[\int_t^{+\infty} e^{-\rho(s-t)} d \; dt\right] = \frac{d}{\rho}$$

and the Regulator's optimal intervention trigger:

$$x_b := \frac{\beta(c+d)(\rho-\mu)}{(\beta-1)(1-\xi)\rho}$$

Because of assumption (2), depositors know if the value of the assets' proceedings under bankruptcy are sufficient to cover their claims. Therefore, for the bank, it suffices to set up a level of non-deposit claims that, at the trigger x_b (selected by the regulator), secures that the present value of depository payments are fully covered. Specifically, I set:

$$(1-\epsilon)\frac{x_b}{\rho-\mu} \geq \frac{d}{\rho}$$

which, after rearranging, yields the no bank-run condition in equation (17).

Consider the parameters that define the financial institution: (i) the growth rate of cash-flows (μ), (ii) the uncertainty of cash-flows (σ), and (iii) the bankruptcy costs (ϵ). Taking first order derivatives on γ:

$$\begin{aligned}
\frac{\mathrm{d}\gamma}{\mathrm{d}\mu} &= \tfrac{1-\xi}{1-\epsilon}\tfrac{\mathrm{d}}{\mathrm{d}\mu}\left(\tfrac{\beta(\mu)-1}{\beta(\mu)}\right) < 0, \quad \forall \mu \in]-\infty, \rho[\\
\frac{\mathrm{d}\gamma}{\mathrm{d}\sigma} &= \tfrac{1-\xi}{1-\epsilon}\tfrac{\mathrm{d}}{\mathrm{d}\mu}\left(\tfrac{\beta(\sigma)-1}{\beta(\sigma)}\right) > 0, \quad \forall \sigma \in]0, +\infty[\\
\frac{\mathrm{d}\gamma}{\mathrm{d}\epsilon} &= \tfrac{(\beta-1)(1-\xi)}{\beta}\tfrac{\mathrm{d}}{\mathrm{d}\epsilon}\left(\tfrac{1}{1-\epsilon}\right) > 0, \quad \forall \epsilon \in]0, 1[
\end{aligned}$$

I also take the first derivative of the multiplier of non-deposit liabilities over deposits to the capital requirement buffer ratio ξ, I have:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\xi} = \frac{(\beta-1)}{\beta(1-\epsilon)}\frac{\mathrm{d}}{\mathrm{d}\xi}(1-\xi) < 0, \quad \forall \xi \in]0, \bar{\xi}_1[$$

Therefore, banks with simultaneous low or negative growth, high volatility, and high bankruptcy costs will have to hold a higher ratio of non-deposit coupons over deposit coupons than banks with high growth, low volatility, and low bankruptcy costs. Moreover, higher □

Proof of Proposition (2). The initial optimisation problem is

$$\begin{aligned}
(c^*, d^*, x_b^*) \quad & \in \arg\max_{\{c,d\}} B(x_0) \\
\text{s.t.} \quad & x_b := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{d+c}{\rho} && \text{[Closure Trigger]} \\
& c \geq d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right) && \text{[NBR Condition]} \\
& x_0 \geq \frac{(\rho-\mu)(c+d)}{\rho(1-\xi)} && \text{[Rationality Constraint]} \\
& c \geq 0 \quad , \quad d \geq 0 && \text{[Non-Negative Constraints]}
\end{aligned}$$

I must restrict the problem to the feasible cases. First notice that I have two major cases: Case A when $\xi < \bar{\xi}$ and Case B when $\xi \geq \bar{\xi}$. For Case A, the no bank-run condition (equation (17)) eliminates the lowest firm value outcome case in which depositors are not fully paid. Case B however is not affected by the NBR Condition because a capital requirement buffer $\xi \geq \bar{\xi}$ always ensures that all creditors, and hence depositors, are fully paid. I take the NBR constraint into the bank's value function (15) and rewrite it for time $t = 0$ as:

$$B(x_0) = \begin{cases} (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_0}{x_b}\right)^\beta\left(\tau\left(\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right) - \epsilon\frac{x_b}{\rho-\mu}\right) & , \quad \xi < \bar{\xi} \quad \text{[Case A]} \\ (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} - (1-\tau)\left(\frac{x_0}{x_b}\right)^\beta\left(\epsilon\frac{x_b}{\rho-\mu}\right) & , \quad \xi \geq \bar{\xi} \quad \text{[Case B]} \end{cases}$$

▷ **Feasible Set solution.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_1 = \left\{(c,d) \in \mathbb{R}^2 | g(c,d) \leq 0\right\},$$

I have two possible cases: Case A for $\xi < \overline{\xi}$ and Case B for $\xi \geq \overline{\xi}$. Depending on the case, some constraints are not relevant or redundant. In detail, I will have the following vector of constraints:

$$g^A(c,d) = \begin{pmatrix} c + d - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -d \\ -c + d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right) \end{pmatrix} \qquad g^B(c,d) = \begin{pmatrix} c + d - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -d \\ -c \end{pmatrix} \tag{108}$$

Regarding Case A, for a family of constants that secures $m = \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 > 0$, the NBR Constraint leads to $c > 0$ for all points except $(c,d) = (0,0)$ where the NBR constraint and the coupon's non-negative constraints are redundant. Therefore, I can eliminate $c \geq 0$ from the constraint set in Case A. Combining the NBR constraint with the Rationality constraint and $d \geq 0$, I guarantee that $\mathcal{G}_1$ is compact in this first case. In Case B, the NBR constraint is unnecessary because $\xi \geq \overline{\xi}$ always guarantees that NBR is fulfilled. Now, $c \geq 0$ is introduced to secure, jointly with the Rationality Constraint and $d \geq 0$, that $\mathcal{G}_1$ is compact for case B. Consequently, with a compact set $\mathcal{G}_1$ in both cases, the Weierstrass Theorem guarantees the existence of a maximising solution.

▷ **Constraint Qualification.** Consider the Constraint Qualification matrix:

$$CQ_A := \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 \end{pmatrix} \qquad CQ_B := \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{109}$$

In Case A, $\xi < \overline{\xi} \Rightarrow \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 > 0 \Rightarrow rank(CQ_A) = 2$. In Case B, $rank(CQ_B) \leq 2$ as the three conditions cannot bind at the same time. I have full rank for all candidates to binding constraints.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers $(\chi_1^A, \chi_2^A, \chi_3^A)$ for Case A and $(\chi_1^B, \chi_2^B, \chi_3^B)$ for Case B. For Case A, the Lagrangian is written as:

$$\begin{aligned} \mathscr{L}(c,d)|_{\xi<\overline{\xi}} =& (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_0}{x_b}\right)^\beta \left(\tau\left(\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right) - \epsilon\frac{x_b}{\rho-\mu}\right) \\ &+ \chi_1^A\left(\rho(1-\xi)x_0 - (\rho-\mu)(c+d)\right) + \chi_2^A d + \chi_3^A\left(c - d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right)\right) \end{aligned} \tag{110}$$

And for Case B:

$$\begin{aligned} \mathscr{L}(c,d)|_{\xi\geq\overline{\xi}} =& (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} - (1-\tau)\left(\frac{x_0}{x_b}\right)^\beta \left(\epsilon\frac{x_b}{\rho-\mu}\right) \\ &+ \chi_1^B\left(\rho(1-\xi)x_0 - (\rho-\mu)(c+d)\right) + \chi_2^B d + \chi_3^B c \end{aligned} \tag{111}$$

▷ **Kuhn-Tucker Conditions.**

Case A: First Order Conditions are:

$$\frac{dB(x_0)}{dc} = 0 \Rightarrow \qquad \frac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{\rho(\xi-1)}\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^\beta - \frac{\chi_1^A}{\rho} + \chi_3^A + \frac{\tau}{\rho} = 0 \tag{112}$$

$$\frac{dB(x_0)}{dd} = 0 \Rightarrow \quad \frac{\chi_3^A(-\beta\xi+\beta\epsilon+\xi-1)}{\beta(\epsilon-1)} + \frac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{\rho(\xi-1)}\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^\beta - \frac{\chi_1^A}{\rho} + \chi_2^A + \frac{\tau}{\rho} = 0 \tag{113}$$

with complementary slack conditions:

$$\chi_1^A\left(\rho(1-\xi)x_0 - (\rho-\mu)(c+d)\right) = 0 \tag{114}$$

$$\chi_2^A d = 0 \tag{115}$$

$$\chi_3^A\left(c - d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right)\right) = 0 \tag{116}$$

And Lagrange multiplier constraints:

$$\chi_1^A \geq 0 \qquad \chi_2^A \geq 0 \qquad \chi_3^A \geq 0 \tag{117}$$

Only solutions satisfying (114)-(117) are admissible.

Case B: First Order Conditions are:

$$\frac{dB(x_0)}{dc} = 0 \Rightarrow \quad \frac{\beta(\tau-1)\epsilon\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}}{\rho(\xi-1)} - \frac{\chi_1^B}{\rho} + \chi_3^B + \frac{\tau}{\rho} = 0 \tag{118}$$

$$\frac{dB(x_0)}{dd} = 0 \Rightarrow \quad \frac{\beta(\tau-1)\epsilon\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}}{\rho(\xi-1)} - \frac{\chi_1^B}{\rho} + \chi_2^B + \frac{\tau}{\rho} = 0 \tag{119}$$

with complementary slack conditions:

$$\chi_1^B\left(\rho(1-\xi)x_0 - (\rho-\mu)(c+d)\right) = 0 \tag{120}$$

$$\chi_2^B d = 0 \tag{121}$$

$$\chi_3^B c = 0 \tag{122}$$

And Kuhn-Tucker multiplier constraints:

$$\chi_1^B \geq 0 \qquad \chi_2^B \geq 0 \qquad \chi_3^B \geq 0 \tag{123}$$

Only solutions satisfying (120)-(123) are admissible.

▷ **Corner Solutions.** I have six possible candidates to corner solutions for each Case A and B . To distinguish them I use, just for the sake of exposure in this proof, the superscript (ij). Here, $i \in A, B$ is the Case and $j \in \{1, ..., 6\}$ is the solution candidate under study. For example, in the first solution candidate for Case A, I need to find $c_2^{(A1)}$, $d_2^{(A1)}$ and $\chi_1^{(A1)}$.

▷ **Solution Candidates.**

A1. $g_1^A(c, d)$ **is binding, i.e.** $(\chi_1^A > 0, \chi_2^A = \chi_3^A = 0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c^{(A1)}, d^{(A1)})$ and the Kuhn-Tucker multiplier $\chi_1^{(A1)}$. Recalling that $l = c + d$, I have:

$$l^{(A1)} = \frac{\rho(1-\xi)}{\rho-\mu}x_0$$

$$\chi_1^{(A1)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}(\beta\xi\tau - \beta\epsilon - \xi\tau + \tau)}{\xi - 1} + \tau$$

This solution candidate does not allow me to define a full solution for each coupon. However, any par of coupons (c, d) is optimal if it satisfies equation (C), the inequality:

$$c > d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right)$$

To guarantee $\chi_1^{(A1)} > 0$, it suffices to have:

$$\xi > \frac{\tau + \left(\frac{\beta-1}{\beta}\right)^{\beta}(\beta\epsilon - \tau)}{\beta\tau}$$

One way to guarantee $\chi_1^{(A1)} > 0$ is to restrict my tax rate τ to

$$\tau > -\frac{\beta\epsilon\left(\frac{\beta-1}{\beta}\right)^{\beta}}{1-\left(\frac{\beta-1}{\beta}\right)^{\beta}} > 0 \tag{124}$$

where $\left(\frac{\beta-1}{\beta}\right)^{\beta} > 0$. This ensures that any $\xi \in [0, \bar{\xi})$ is feasible.

A2. $g_2^A(d)$ **is binding, i.e.** $(\chi_1^A = 0, \chi_2^A > 0, \chi_3^A = 0)$ In this solution candidate, the Non-Negative Constraint for Deposits is the only constraint binding. I solve for the optimal coupons $(c^{(A2)}, d^{(A2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(A2)}$. I find however that

$$\chi_2^{(A2)} = 0$$

This implies that constraint $g_2^A(d)$ does not bind and the candidate solution is not feasible. However, for a d sufficiently small to be negligible, I have

$$c < \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta} x_0 =: c_1^{(A2)}$$

A3. $g_3^A(c,d)$ is binding, i.e. $(\chi_1^A = 0, \chi_2^A = 0, \chi_3^A > 0)$ I am considering that NBR Constraint binds. The optimisation problem yields an optimal multiplier solution that suggests, however, that the candidate solution never binds.

$$\chi_3^{(A3)} = 0$$

The NBR Constraint is never binding. Nonetheless, I can find the non-binding solution for the NBR Constraint. That boundary is:

$$c > \frac{\rho}{\rho - \mu} \frac{\beta(\epsilon - \xi) + \xi - 1}{\beta} \left(\frac{(\xi - 1)\tau}{\beta(\epsilon - \xi\tau) + (\xi - 1)\tau} \right)^{-1/\beta} x_0 =: c^{(A3)}$$

A4. $g_1^A(c,d)$ and $g_2^A(d)$ bind, i.e. $(\chi_1^A > 0, \chi_2^A > 0, \chi_3^A = 0)$ In this solution candidate, I check the case in which the Non-Negative Constraint for Deposits and the Rationality Constraint bind. This candidate solution does not however hold at the Non-Negative Constraint for Deposits:

$$\chi_2^{(A4)} = 0$$

Even though

$$\chi_1^{(A4)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta} (\beta\xi\tau - \beta\epsilon - \xi\tau + \tau)}{\xi - 1} + \tau$$

Therefore, I must always fulfil the condition:

$$c < \frac{\rho(1-\xi)}{\rho - \mu} x_0 := c^{(A4)}$$

A5. $g_1^A(c,d)$ and $g_3^A(c,d)$ bind, i.e. $(\chi_1^A > 0, \chi_2^A = 0, \chi_3^A > 0)$ I check if binding the Rationality Constraint and the NBR Constraint leads to a feasible candidate solution. However, I find

$$\chi_3^{(A5)} = 0$$

$$\chi_1^{(A5)} = \left(\frac{\beta - 1}{\beta}\right)^{\beta} \frac{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}{\xi - 1} + \tau$$

The null multiplier tells me, as expected, that the NBR condition is not binding. In this case, I have for the total debt coupon:

$$l^{(A5)} = \frac{\rho(1-\xi)}{\rho - \mu} x_0$$

and

$$c > -\frac{\rho(\beta(\epsilon - \xi) + \xi - 1)}{(\beta - 1)(\mu - \rho)} x_0 =: c^{(A5)}$$

A6. $g_2^A(c,d)$ and $g_3^A(c,d)$ bind, i.e. $(\chi_1^A = 0, \chi_2^A > 0, \chi_3^A > 0)$ I check if binding both Non-negative Constraints lead to a feasible candidate solution. This solution is not a feasible candidate, as the multipliers are negative:

$$\chi_2^{(A6)} = -\frac{(\beta - 1)(\xi - 1)\tau}{\beta\rho(\epsilon - 1)} < 0$$

$$\chi_3^{(A6)} = -\frac{\tau}{\rho} < 0$$

B1. $g_1^B(c,d)$ is binding, i.e. $(\chi_1^B > 0, \chi_2^B = \chi_3^B = 0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c^{(B1)}, d^{(B1)})$ and the Kuhn-Tucker multiplier $\chi_1^{(B1)}$. Recalling that $l_1 = c + d$, I have:

$$l^{(B1)} = \frac{\rho(1-\xi)}{\rho - \mu} x_0$$

$$\chi_1^{(B1)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta} (1-\tau)\beta\epsilon}{1 - \xi} + \tau > 0$$

This solution candidate does not allow me to define a full solution for each coupon. However, any par of coupons (c, d) is optimal if it satisfies equation (C).

B2. $g_2^B(d)$ is binding, i.e. $(\chi_1^B = 0, \chi_2^B > 0, \chi_3^B = 0)$ In this solution candidate, the Non-Negative Constraint for Deposits is the only constraint binding. I solve for the optimal coupons $(c^{(B2)}, d^{(B2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(B2)}$. I find however that

$$\chi_2^{(B2)} = 0$$

This implies that constraint $g_2^B(d)$ does not bind and the candidate solution is not feasible. However, for a d sufficiently small to be negligible, I have

$$c < \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0 =: c_1^{(B2)}$$

B3. $g_3^B(c,d)$ is binding, i.e. $(\chi_1^B = 0, \chi_2^B = 0, \chi_3^B > 0)$ I am considering that the Non-Negative Constraint for Non-Deposit Liabilities binds. The optimisation problem yields an optimal multiplier where the candidate solution never binds.

$$\chi_3^{(B3)} = 0$$

The Non-Negative Constraint is never binding. Nonetheless, I can find the non-binding solution for the Non-Negative Constraint. That boundary is:

$$c > 0 =: c^{(B3)}$$
$$d < \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0 =: d^{(B3)}$$

B4. $g_1^B(c,d)$ and $g_2^B(d)$ bind, i.e. $(\chi_1^B > 0, \chi_2^B > 0, \chi_3^B = 0)$ In this solution candidate, I check the case in which the Non-Negative Constraint for Deposits and the Rationality Constraint bind. This candidate solution does not however hold at the Non-Negative Constraint for Deposits:

$$\chi_2^{(B4)} = 0$$

Even though

$$\chi_1^{(B4)} = \frac{\beta\left(\frac{\beta-1}{\beta}\right)^{\beta}(\tau-1)\epsilon}{\xi-1} + \tau > 0$$

The following conditions must hold close to the aforementioned boundaries:

$$c < \frac{\rho(1-\xi)}{\rho-\mu}x_0 := c^{(B4)}$$
$$d > 0 =: d^{(B4)}$$

B5. $g_1^B(c,d)$ and $g_3^B(c,d)$ bind, i.e. $(\chi_1^B > 0, \chi_2^B = 0, \chi_3^B > 0)$ I check if binding the Rationality Constraint and the Non-negative Constraint for Non-Deposit Liabilities lead to a feasible candidate solution. However, I find

$$\chi_3^{(B5)} = 0$$
$$\chi_1^{(B5)} = \frac{\beta\left(\frac{\beta-1}{\beta}\right)^{\beta}(\tau-1)\epsilon}{\xi-1} + \tau > 0$$

The null multiplier tells me, as expected, that the Non-negative Constraint is not binding. In this case, I must always have a positive level of non-deposit liabilities. As for Case A, the total debt coupon:

$$l^{(B5)} = \frac{\rho(1-\xi)}{\rho-\mu}x_0$$

and the coupons must respect

$$c > 0 =: c^{(B5)}$$
$$d < \frac{(\xi-1)\rho x_0}{\mu-\rho} =: d^{(B5)}$$

B6. $g_2^B(c,d)$ and $g_3^B(c,d)$ bind, i.e. $(\chi_1^B = 0, \chi_2^B > 0, \chi_3^B > 0)$ I check if binding both Non-negative Constraints lead to a feasible candidate solution. This solution is not a feasible candidate, as the multipliers are negative:

$$\chi_2^{(B6)} = -\frac{\tau}{\rho} < 0$$
$$\chi_3^{(B6)} = -\frac{\tau}{\rho} < 0$$

▷ **Interior Solution $(\chi_1^A = \chi_2^A = \chi_3^A = 0)$ and $(\chi_1^B = \chi_2^B = \chi_3^B = 0)$ - First Order Conditions.** In this case, the solution for the first order result cannot specify a specific optimal solution for each type of liability but only for the total debt coupon. Recall from the beginning that I define $l = c + d$ as the periodic total coupon to be paid.

Case A

So, any pair (c,d) is optimal if $c^{**} + d^{**} = l^{**}$, with:

$$l^{**} = \frac{(\beta-1)(1-\xi)\rho\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0 \tag{125}$$

Case B

Case B.1 $(\chi_1^B = \chi_2^B = \chi_3^B = 0)$. In this case:

$$l^{**} = \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0 \tag{126}$$

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm $B(x_0)$ is concave for the unconstrained situation. This is relevant to guarantee interior solutions are maximising arguments.

Case A: Taking second order conditions and rearranging, I guarantee that l^{**} is a maximising argument for the firm value function conditional on subset $\xi \in [0, \bar{\xi})$ iff:

$$\xi < \frac{\beta\epsilon - \tau}{(\beta-1)\tau}$$

However, since Case A is conditional on subset $\xi \in [0, \bar{\xi})$, I can note that $\frac{\beta\epsilon-\tau}{(\beta-1)\tau} > \bar{\xi}$ always for $\tau < 1$. Given that the tax rate $\tau \in (0,1)$, then SOC is always negative:

$$\underbrace{\beta}_{<0} \underbrace{\left(\frac{x_0}{x_b}\right)^{\beta} \frac{1}{\rho(1-\xi)l^{**}}}_{>0} \underbrace{((1-\xi)\tau - \beta(\epsilon - \xi\tau))}_{>0} < 0$$

Case B: Taking second order conditions and rearranging, I guarantee that l_1^{**} is a maximising argument for the firm value function conditional on subset $\xi \in [\bar{\xi}]$:

$$\underbrace{-\frac{\beta^2(1-\tau)\epsilon}{\rho(1-\xi)l^{**}}}_{<0} \underbrace{\left(\frac{x_0}{x_b}\right)^{\beta}}_{>0} < 0$$

▷ **Equilibrium Solution.** By solving the constrained optimisation problem, I can synthesise the main equilibrium result. First, let me denominate some of the key parameters as:

$$\bar{c} := \begin{cases} c^{(A4)} & , \quad \xi < \bar{\xi} \\ c^{(B4)} & , \quad \xi \geq \bar{\xi} \end{cases} \qquad \hat{c} := \begin{cases} c^{(A5)} & , \quad \xi < \bar{\xi} \\ c^{(B5)} & , \quad \xi \geq \bar{\xi} \end{cases}$$

$$\underline{c} := \begin{cases} c^{(A3)} & , \quad \xi < \bar{\xi} \\ c^{(B3)} & , \quad \xi \geq \bar{\xi} \end{cases} \qquad \tilde{c} := \begin{cases} c^{(A2)} & , \quad \xi < \bar{\xi} \\ c^{(B2)} & , \quad \xi \geq \bar{\xi} \end{cases}$$

$$c^{**} := \begin{cases} c^{(A3)} & , \quad \xi < \bar{\xi} \\ c^{(B3)} & , \quad \xi \geq \bar{\xi} \end{cases}$$

and, taking the Rationality Constraint at its binding point, let me write:

At time $t = 0$, the bank selects the coupons (c^*, d^*) to be paid every time $t \in [0, T_b[$. The total periodic coupon $l^* = c^* + d^*$ is optimally defined as:

$$l^* = \min\left\{l^{**}, \bar{l}\right\} \tag{127}$$

where

$$l^{**} = \begin{cases} \frac{(\beta-1)(1-\xi)\rho\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi < \overline{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi \geq \overline{\xi} \end{cases} \tag{128}$$

and

$$\bar{l} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{129}$$

I cannot find a specific solution for coupons (c^*, d^*). However, I let the deposits coupon be defined as the difference between the total coupon and the non-deposit liabilities coupon and define the interval of feasible solutions for the non-deposit liabilities coupons. Therefore, I have:

$$d^* := l^* - c^* \qquad \text{and} \qquad c^* \in \begin{cases}]\hat{c}, \overline{c}[& , \text{ for } l^{**} \geq \bar{l} \\ ]\underline{c}, \tilde{c}[& , \text{ for } l^{**} < \bar{l} \end{cases} \tag{130}$$

where $\hat{c}$ is the lower boundary for the optimal c^*. In this case, the equilibrium l^* binds at the Rationality Constraint and is weakly bounded by the No Bank-Run Condition if $\xi < \overline{\xi}$ (Case A), and by the Non-negative constraint for Non-Deposit Liabilities if $\xi \geq \overline{\xi}$ (Case B). $\hat{c}$ is written as:

$$\hat{c} = \begin{cases} -\frac{\rho(\beta(\epsilon-\xi)+\xi-1)}{(\beta-1)(\mu-\rho)} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{131}$$

$\overline{c}$ is the upper boundary when l^* binds at the Rationality Constraint and c^* is weakly bounded by the Non-negative constraint for Deposits. $\overline{c}$ is written as:

$$\overline{c} = \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{132}$$

$\underline{c}$ is the lower bound when the interior solution l^{**} holds and is weakly bounded by the No Bank-Run Condition if $\xi < \overline{\xi}$ (Case A), and by the Non-negative constraint for Non-Deposit Liabilities if $\xi \geq \overline{\xi}$ (Case B).

$$\underline{c} = \begin{cases} \frac{\rho}{\rho-\mu} \frac{\beta(\epsilon-\xi)+\xi-1}{\beta} \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{133}$$

$\tilde{c}$ is the upper boundary when the interior solution l^{**} holds and c^* is weakly bounded by the Non-negative constraint for Deposits. $\tilde{c}$ is written as:

$$\tilde{c} = \begin{cases} \frac{\beta-1}{\beta} \frac{\rho}{\rho-\mu} (1-\xi) \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{134}$$

□

D Appendix Costless Deposit Insurance

Proof of Lemma 3. Recall the present value of all future payments to depositors d/ρ and the optimal closure trigger (31) at the threshold $\underline{\xi}$:

$$x_{di}(\underline{\xi}) := \frac{\beta(c_{di} + d_{di})(\rho-\mu)}{(\beta-1)(1-\underline{\xi})\rho}$$

Then solving $(1-\epsilon)\frac{x_{di}(\underline{\xi})}{\rho-\mu} = \frac{d}{\rho}$ for $\underline{\xi}$ yields equation (40). □

Proof of Proposition 3. Before solving the equilibrium, I can refine the problem by excluding the cases already solved in the base case. In the presence of a DIS, I have three branches possible depending on the exogenous parameter ξ.

▷ **.** I divide the bank's value function $B_{di}(x_t)$ at the lower threshold. I call $B_{di}(x_t|\xi \leq \underline{\xi})$ when the bankruptcy proceedings cannot fully cover all deposits and $B_{di}(x_t|\xi > \underline{\xi})$ when the bankruptcy proceedings can always cover all deposits. In my setup it is the case that

$$B_{di}(x_t|\xi > \underline{\xi}) := B(x_t)$$

For the cases in which depositors are always covered, the value function of the financial institution in the presence of a DIS is the same as in the case without deposit insurance which results in the following remark:

Remark 1. *The equilibria for $\xi > \underline{\xi}$ in the presence of a DIS is the same as in the base case, i.e.*

$$(c^*_{di}, d^*_{di}, x^*_{di}) = (c^*, d^*, x^*_b), \qquad \forall \xi \in]\underline{\xi}, 1] \tag{135}$$

Hence, I restrict to study the optimisation problem for $B_{di}(x_t|\xi \leq \underline{\xi})$. In this case the value functions for Equity-holders, Non-Deposit Creditors and Depositors are respectively:

$$E_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} (1-\tau)\left[\frac{x_t}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right] - \left(\frac{x_t}{x_{di}}\right)^\beta (1-\tau)\left(\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right) & , \text{for } x_t > x_{di} \\ 0 & , \text{for } x_t \leq x_{di} \end{cases}$$

$$C_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} \frac{c_{di}}{\rho} - \left(\frac{x_t}{x_{di}}\right)^\beta \frac{c_{di}}{\rho} & , \text{for } x_t > x_{di} \\ 0 & , \text{for } x_t \leq x_{di} \end{cases} \qquad \text{and} \qquad D_{di}(\xi < \underline{\xi}) = \frac{d_{di}}{\rho}$$

Summing each claimers value functions I obtain the bank's total value is then:

$$B_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{di}+d_{di}}{\rho} - \left(\frac{x_t}{x_{di}}\right)^\beta \left((1-\tau)\left(\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\right) + \tau\frac{c_{di}}{\rho}\right) & , \text{for } x_t > x_{di} \\ \frac{d_{di}}{\rho} & , \text{for } x_t \leq x_{di} \end{cases} \tag{136}$$

Moreover, by inspecting the bank's value function in (136), I see that any marginal increase in value can be performed by deposits instead of non-deposit liabilities at a lower functional cost. I can make an additional simplification for this optimisation problem:

Remark 2. *For $\xi < \underline{\xi}$, only solutions that bind at the non-deposit creditors non-negative constraint are feasible. Formally:*

$$c_{di} = 0$$

▷ **Optimisation Problem.** Taking into consideration Remarks (1) and (2), I rewrite the optimisation problem for the bank as:

$$\begin{aligned} (c^*_{di}, d^*_{di}) \quad & \in \underset{\{c_{di}, d_{di}\}}{\arg\max}\ B_{di}(x_0|\xi \leq \underline{\xi}) & \\ \text{s.t.} \quad & x_{di} := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_{di}+d_{di}}{\rho} & \text{[Closure Trigger]} \\ & x_0 \geq \frac{(\rho-\mu)(c_{di}+d_{di})}{\rho(1-\xi)} & \text{[Rationality Constraint]} \\ & c_{di} = 0 \quad , \quad d_{di} \geq 0 & \text{[Non-Negative Constraints]} \end{aligned}$$

▷ **Constraint Set Feasibility.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_{di} = \left\{(c_{di}, d_{di}) \in \mathbb{R}^2 | g(c_{di}, d_{di}) \leq 0\right\}$$

with

$$g(c_{di}, d_{di}) = \begin{pmatrix} g_1(c_{di}, d_{di}) \\ g_2(d_{di}) \\ g_3(c_{di}) \end{pmatrix} = \begin{pmatrix} c_{di} + d_{di} - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -d_{di} \\ 0 \end{pmatrix}$$

Let $Q_1 := \frac{(1-\xi)\rho}{\rho-\mu}x_0$ be a family of constants and let (ρ, μ, x_0, ξ) be such that $Q_1 > 0$. Then, I secure the feasible set $\mathcal{G}_{di}$ is compact which, by the Weierstrass Theorem, guarantees the existence of a maximising solution.

▷ **Nondegenerate Constraint Qualification.** Consider the Constraint Qualification matrix

$$CQ := \nabla g(c_{di}, d_{di}) = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{137}$$

For any possible combinations of binding constraints, $Rank(CQ) \in \{1, 2\}$. Therefore, I can exclude the case where three constraints are binding. I can imediately see that, in this case, I have two possible solutions either an interior solution for d^*_{di} (with $c^*_{di} = 0$) or a corner solution where d^*_{di} binds at the rationality constraint. A solution with $(c_{di}, d_{di}) = (0, 0)$ is excluded.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers $(\chi_1^C, \chi_2^C, \chi_3^C)$ and take first order conditions.

$$\begin{aligned}\mathscr{L}(c_{di}, d_{di})|_{\xi \leq \underline{\xi}} =& (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{di}+d_{di}}{\rho} - \left(\frac{x_t}{x_{di}}\right)^{\beta}\left((1-\tau)\left(\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\right) + \tau\frac{c_{di}}{\rho}\right) \\ &+ \chi_1^C\left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{di}+d_{di})\right) + \chi_2^C d_{di} + \chi_3^C c_{di}\end{aligned} \tag{138}$$

▷ **Kuhn-Tucker Conditions.**

$$\chi_1^C\left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{di}+d_{di})\right) = 0 \tag{139}$$

$$\chi_2^C d_{di} = 0 \tag{140}$$

$$c_{di} = 0 \tag{141}$$

and Kuhn-Tucker multiplier constraints are:

$$\chi_1^C \geq 0 \quad , \qquad \chi_2^C \geq 0 \tag{142}$$

▷ **Solutions Candidates.** Only solutions satisfying (139)-(142) are admissible. Hence, I reduce to two possible candidates of admissible solutions.

C1. Interior Solution $(\chi_1^C = 0, \chi_2^C = 0)$. For this solution candidate, no inequality constraint will bind. I have:

$$d_{di}^{(C1)} = \frac{(\beta-1)(\xi-1)\rho x_0\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)}$$

with

$$\chi_3^{(C1)} = \frac{\tau-\xi\tau}{\rho(\tau-1)((\beta-1)\xi+1)}$$

The solution multiplier is always $\chi_3^{(C1)} > 0$ because the Second Order Condition for the interior solution requires $\xi > 1/(1-\beta)$.

C2. Corner Solution $(\chi_1^C = 0, \chi_2^C > 0)$. For this solution candidate, the Rationality Constraint binds. I have:

$$d_{di}^{(C2)} = \frac{(1-\xi)\rho x_0}{\rho-\mu}$$

The solution multipliers are:

$$\chi_1^{(C2)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}(1-\tau)((\beta-1)\xi+1)}{1-\xi} + \tau$$

$$\chi_3^{(C2)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}}{\rho} > 0 \quad \text{(always)}$$

I require that $\chi_1^{(C2)} > 0$ for the solution to hold.

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm the concavity of $B_{di}(x_0|\xi < \underline{\xi})$ for the unconstrained situation. Taking second order conditions and rearranging, I guarantee d^*_{bi} is a maximising arguments for the firm value function conditional on subset $\xi \in [0, \underline{\xi})$. The SOC condition is:

$$\left.\frac{\mathrm{d}B_{bi}(x_0|\xi \leq \underline{\xi})}{\mathrm{d}d}\right| < 0 \Leftrightarrow \underbrace{-\beta\frac{1-\tau}{1-\xi}\frac{1}{d^*_{di}}}_{>0}\underbrace{\left(\frac{x_0}{x_{di}}\right)^{\beta}}_{>0}\underbrace{(1+(\beta-1)\xi)}_{<0} \qquad \text{iff} \qquad \xi > \frac{1}{\beta-1}$$

To secure that the interior solution is a maximum, I must impose $\xi > \frac{1}{1-\beta}$.

▷ **Equilibrium Capital Requirement Ratio Threshold.** At last, take into consideration that $c_1^* = 0$. Replacing in $\underline{\xi}(c_{di}, d_{di})$ I see:

$$\underline{\xi}(c^*_{di}, d^*_{di}) = \overline{\xi}$$

In summary, for all $\xi < \overline{\xi}_1$, if the regulator introduces a costless deposit insurance scheme in Scenario 1 (without resolution tools), the bank will preempt the intervention decision by selecting a level of liabilities that is made of 100% deposits. If $\xi \geq \overline{\xi}_1$, the equilibrium choice of debt follows Condition (127), the equilibrium result for Scenario 1 without DIS.

▷ **Equilibrium Coupon.** At time $t = 0$, the bank selects the coupons (c^*_{di}, d^*_{di}) to be paid every time $t \in [0, T_{di}[$. As mentioned in Remark (1), for $\xi > \bar{\xi}$, the equilibrium does not change with the introduction of a DIS. Henceforth, all equilibrium results for $\xi > \bar{\xi}$ correspond to Case B in the proof of Proposition (2). Combining all equilibria, the total periodic coupon $l^*_{di} = c^*_{di} + d^*_{di}$ can be defined as:

$$l^*_{di} = \min\left\{l^{**}_{di}, \bar{l}_{di}\right\} \tag{143}$$

where the interior solution is

$$l^{**}_{di} = \begin{cases} \dfrac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0, & \text{for } \xi < \bar{\xi} \\ \dfrac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi \geq \bar{\xi} \end{cases} \tag{144}$$

and the corner solution (binding at the rationality constraint) is:

$$\bar{l}_{di} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{145}$$

Any combination of $(c^*_{di} d^*_{di})$ is a feasible optimal solution, provided that it respects the following conditions:

$$d^*_{di} := l^*_{di} - c^*_{di} \qquad \text{and} \qquad c^*_{di} = \begin{cases} 0 & , \quad \xi < \bar{\xi} \\ c^{**}_{di} & , \quad \xi \geq \bar{\xi} \end{cases} \tag{146}$$

with

$$c^{**}_{di} \in \begin{cases} \left]0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right[& , \quad \text{for} \quad l^{**} \geq \bar{l} \\ \\ \left]0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right[& , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{147}$$

□

Proof of Proposition 4. I compare the optimal total coupon for the DIS case in condition (44) with the optimal total coupon for the base case in condition (20). In Case (A) I study the equilibria for weak banks and in Case (B) I study equilibria for strong banks.

Remark 3. *For $\beta < 0$, a function $f : z \mapsto z^{\frac{1}{\beta}} - 1$ is always positive in the domain $z \in [0, 1)$.*

▷ **Case (A):** $\xi \in [0, \bar{\xi})$**.** Consider a transition from interior to interior solution. That is:

$$(1) \quad l^* = l^{**} \quad \text{and} \quad l^*_{di} = l^{**}_{di}$$

Writing the ratio between the new equilibrium and the old equilibrium total coupon, I have:

$$\frac{l^{**}_{di}}{l^{**}} := \left(\frac{(\tau-1)((\beta-1)\xi+1)}{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}\right)^{\frac{1}{\beta}}, \qquad \forall \xi < \bar{\xi} \tag{148}$$

Rearranging condition (148) and using Remark 3, the ratio l^{**}_{di}/l^{**} is greater than 1 iff:

$$\beta\epsilon < (1 + (\beta-1)\xi)$$

which holds for all $\xi < \bar{\xi}$. Therefore the interior solution for the equilibrium total coupon always increases when the regulator introduces the costless deposit insurance. That is

$$\frac{l^{**}_{di}}{l^{**}} > 1, \qquad \forall \xi < \bar{\xi} \tag{149}$$

With this result, and given that, the equilibrium total coupon conditions are $l^* = \{l^{**}, \bar{l}\}$ and $l^*_{di} = \{l^{**}_{di}, \bar{l}\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l^{**} < \bar{l} < l^{**}_{di} \quad \Rightarrow \quad l^* = l^{**} \quad \text{and} \quad l^*_{di} = \bar{l} \\ &(3) \quad \bar{l} < l^{**} < l^{**}_{di} \quad \Rightarrow \quad l^* = \bar{l} \quad \text{and} \quad l^*_{di} = \bar{l} \end{aligned} \tag{150}$$

Consider Scenario (2): $l^* = l^{**}$ and $l^*_{di} = \bar{l}$. Since it must be the case $l^{**} < \bar{l} < l^{**}_{di}$, the result presented in condition (149) guarantees that the following condition always holds:

$$\frac{\bar{l}_{di}}{l^{**}} = \frac{\beta}{\beta - 1}\left(\frac{\tau(1-\xi)}{-\beta\epsilon + \tau(1 + (\beta - 1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi < \bar{\xi} \tag{151}$$

In Scenario (3), the optimal coupon does not change: $l^* = \bar{l}$ and $l^*_{di} = \bar{l}$.

▷ **Case (B):** $\xi \in [\bar{\xi}, 1]$**.** Comparing condition (44) with the condition (20) we see that the total coupon does not change:

$$l^*_{di} = l^*, \qquad \forall \xi \geq \bar{\xi}$$

The financial institution has no incentive to change its equilibrium total leverage coupon if deposit insurance is introduced.

□

Proof of Proposition 5. Included in the Proof of Proposition (3). □

Proof of Proposition 6. Let me represent the absolute change in the claims with a Δ. The change in equilibrium bank value after the introduction of costless deposit insurance is

$$\Delta B_{di}(x|\mathcal{E}_{di}) = B_{di}(x|\mathcal{E}_{di}) - B(x|\mathcal{E}_{di})$$

Furthermore, the change in equity value is $\Delta E_{di}(x|\mathcal{E}_{di})$ when I introduce the costless deposit insurance, the change in non-deposit liabilities is $\Delta C_{di}(x|\mathcal{E}_{di})$ and the change in deposits is $\Delta D_{di}(\mathcal{E}_{di})$. All these represent the change in the claims value function from the base case to the costless deposit insurance case. Moreover, note that

$$\Delta B_{di}(x|\mathcal{E}_{di}) = \Delta E_{di}(x|\mathcal{E}_{di}) + \Delta C_{di}(x|\mathcal{E}_{di}) + \Delta D_{di}(\mathcal{E}_{di})$$

For the remaining of this proof I will consider equilibria will hold respectively in the base and DIS case and, so, I omit the equilibrium notation, e.g. $B_{di}(x) \equiv \Delta B_{di}(x|\mathcal{E}_{di})$.

Furthermore, consider the following Remarks:

Remark 4. *A function $f : (l, \beta) \mapsto l^{1-\beta}$ with $l \in \mathbb{R}_+$ and $\beta \in \mathbb{R}_-$ is monotonically increasing in l.*

Remark 5. *I restrict to the tuple of parameters (μ, σ, ρ, ξ) that secures $\xi > \frac{1}{1-\beta}$*

These remarks aim at easing the present proof. Remark (4) characterises a function that appears recurrently and Remark (5) imposes a necessary condition in my parameterisation. I note that this last remark is necessary to obtain maximising solutions in the costless deposit insurance case, as demonstrated in the Proof of Proposition (3).

▷ **Capital structure when** $\xi \geq \bar{\xi}$**.** I can see that the model does not change if the bank is positioned in $\xi \geq \bar{\xi}$ by comparing the value function (38), for the deposit insurance case, with value function (15) for the base case. Moreover, this is also the case for the equity value function (34) and (11), respectively. Hence, using Result (**??**), I have:

$$\Delta E_{di}(x|\xi \geq \bar{\xi}) = 0 \qquad \text{and} \qquad \Delta B_{di}(x|\xi \geq \bar{\xi}) = 0$$

which leads to

$$\Delta CS_{di}(x|\xi \geq \bar{\xi}) = 0$$

▷ **Capital structure when** $\xi < \bar{\xi}$**.** Let me write the value functions for this subcase. The difference value function for equity in equilibrium as:

$$\begin{aligned} &\Delta E_{di}(x|\xi < \bar{\xi}) := E_{di}(x|\xi < \bar{\xi}) - E(x|\xi < \bar{\xi}) = \\ &= \frac{1-\tau}{\rho}(l^* - l^*_{di}) + \frac{1-\tau}{\rho}\left(\frac{x}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}}\right)^{\beta}\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi} - 1\right)\left((l^*)^{1-\beta} - (l^*_{di})^{1-\beta}\right) \end{aligned} \tag{152}$$

Given Result (**??**) and Remark (4) it is easy to show that $\Delta E_{di}(x|\xi < \bar{\xi}) \leq 0$ but I will discuss this next in greater detail for each one of the possible cases. Moreover, the difference value function for total bank value is:

$$\begin{aligned} &\Delta B_{di}(x|\xi < \bar{\xi}) := B_{di}(x|\xi < \bar{\xi}) - B(x|\xi < \bar{\xi}) = \\ &= \frac{\tau}{\rho}(l^*_{di} - l^*) - \frac{x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}{\rho(1-\beta)(1-\xi)}\left((l^*_{di})^{1-\beta}(\tau-1)((\beta-1)\xi+1) - (l^*)^{1-\beta}(-\beta\epsilon + \tau(1+(\beta-1)\xi))\right) \end{aligned} \tag{153}$$

Now, considering Result (??) and given that $\bar{l}$ is an upperbound for interior solutions l^{**}_{di} or l^{**}, I restrict my analysis to three cases:

$$\begin{array}{llcll} \text{(A)} & \bar{l} < l^{**} < l^{**}_{di} & \Rightarrow & l^* = \bar{l} \quad \text{and} \quad l^*_{di} = \bar{l} \\ \text{(B)} & l^{**} < \bar{l} < l^{**}_{di} & \Rightarrow & l^* = l^{**} \quad \text{and} \quad l^*_{di} = \bar{l} \\ \text{(C)} & l^{**} < l^{**}_{di} < \bar{l} & \Rightarrow & l^* = l^{**} \quad \text{and} \quad l^*_{di} = l^{**}_{di} \end{array} \tag{154}$$

Case (A): $\bar{l} < l^{**} < l^{**}_{di}$**.** In this case, the tuple of parameters $(\mu, \sigma, \rho, \tau, \xi, \epsilon)$ is such that, in the base case and in the costless deposit insurance case, the bank always chooses a level of total coupon that binds at the rationality constraint. Specifically, $l^* = \bar{l}$ for the base case and $l^*_{di} = \bar{l}$ after I introduce the costless deposit insurance.

Therefore, I have:

$$\Delta E^{(A)}_{di}(x|\xi < \bar{\xi}) = 0 \tag{155}$$

and

$$\Delta B^{(A)}_{di}(x|\xi < \bar{\xi}) := \underbrace{-\frac{x^\beta \left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta} \bar{l}^{1-\beta}}{\rho(1-\beta)(1-\xi)}}_{<0} \underbrace{((\tau-1)((\beta-1)\xi+1) - (-\beta\epsilon + \tau(1+(\beta-1)\xi)))}_{\substack{<0 \\ \text{by } \xi < \bar{\xi}}} > 0 \tag{156}$$

As a result, the capital structure decreases due to an increase in the denominator, i.e.

$$\Delta CS^{(A)}(x|\xi < \bar{\xi}) < 0$$

Case (B): $l^{**} < \bar{l} < l^{**}_{di}$**.** In this case, the tuple of parameters $(\mu, \sigma, \rho, \tau, \xi, \epsilon)$ is such that, in the base case the bank always chooses an interior solution for the optimal level of total coupon and in the costless deposit insurance case the bank's optimal total coupon binds at the rationality constraint. In detail, $l^* = l^{**}$ for the base case and $l^*_{di} = \bar{l}$ for the costless deposit insurance case.

Using condition (151) with Remarks (4) and (5), I have:

$$\Delta E^{(B)}_{di}(x|\xi < \bar{\xi}) = \underbrace{\frac{1-\tau}{\rho}\left(l^{**} - \bar{l}\right)}_{<0} + \underbrace{\frac{1-\tau}{\rho}\left(\frac{x}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}}\right)^\beta}_{>0} \underbrace{\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi} - 1\right)}_{\substack{>0 \\ \text{by Remark (5)}}} \underbrace{\left((l^{**})^{1-\beta} - (\bar{l})^{1-\beta}\right)}_{\substack{<0 \\ \text{by Remark (4)}}} < 0 \tag{157}$$

and

$$\Delta B^{(B)}_{di}(x|\xi < \bar{\xi}) = \underbrace{\frac{\tau}{\rho}\left(\bar{l} - l^{**}\right)}_{>0} - \underbrace{-\frac{x^\beta \left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}{\rho(1-\beta)(1-\xi)}}_{<0} \underbrace{\left(\left(\bar{l}\right)^{1-\beta}(\tau-1)((\beta-1)\xi+1) - (l^{**})^{1-\beta}(-\beta\epsilon + \tau(1+(\beta-1)\xi))\right)}_{<0} > 0 \tag{158}$$

In case B, the capital structure decreases due to a simultaneous decrease in the numerator (equity value function) and an increase in the denominator (bank's value function), i.e.

$$\Delta CS^{(B)}(x|\xi < \bar{\xi}) < 0$$

Case (C): $l^{**} < l^{**}_{di} < \bar{l}$**.** In this case, the tuple of parameters $(\mu, \sigma, \rho, \tau, \xi, \epsilon)$ is such that, in the base case and in the costless deposit insurance the bank always chooses an interior solution. In this case, $l^* = l^{**}$ for the base case and $l^*_{di} = l^{**}_{di}$ for the costless deposit insurance case. Using condition (148) with Remarks (4) and (5), I have:

$$\Delta E_{di}(x|\xi < \bar{\xi}) = \underbrace{\frac{1-\tau}{\rho} l^{**}_{di}\left(1 + \left(\frac{x}{x_{di}}\right)^\beta \left(\frac{\beta}{\beta-1}\frac{1}{1-\xi} - 1\right)\right)}_{>0} \underbrace{\left(\left(\frac{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{\frac{1}{\beta}} - 1\right)}_{\substack{<0 \\ \text{by Condition (148)}}} < 0 \tag{159}$$

and

$$\Delta B_{di}(x|\xi < \bar{\xi}) = \underbrace{\frac{l^{**}_{di}}{\rho}\left(1 - \left(\frac{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{\frac{1}{\beta}}\right)}_{>0} \underbrace{\left(\tau - \left(\frac{x}{x_{di}}\right)^\beta \frac{(\tau-1)((\beta-1)\xi+1)}{(1-\beta)(1-\xi)}\right)}_{>0} > 0 \tag{160}$$

In case C, the capital structure decreases due to a simultaneous decrease in the numerator (equity value function) and an increase in the denominator (bank's value function), i.e.

$$\Delta CS^{(C)}(x|\xi < \bar{\xi}) < 0$$

□

E Appendix of Private Sector Acquisitions Case

Proof of Lemma 4. This Proof is identical to the Proof of Lemma 1. □

Proof of Proposition 7. This Proof is identical to the Proof of Proposition 2 provided ϵ is replaced with ϵ_p. □

Proof of Proposition 8. Consider three cases referring each partition of the bank's type: Case (A) corresponds to the weakest banks, the group of institutions that respect the condition $\xi < \bar{\xi}_p$. Case (B) corresponds to the intermediate region, i.e. the institutions that were once deemed as weak but, with the introduction of the PSA, they are now considered strong. Specifically, $\xi \in [\bar{\xi}_p, \bar{\xi}]$. At last, Case (C) corresponds to the banks that were always strong, i.e. $\xi \geq \bar{\xi}$.

▷ **Case (A):** $\xi \in [0, \bar{\xi}_p)$**.** In this case the bank's value function and payoff structure never changes. The proceedings from the bank's closure will continue to be insufficient to cover liabilities and, in this circumstance, the bank benefits from deposit insurance which it will explore by selecting a maximum level of deposits in the liability structure. Hence, for $\xi < \bar{\xi}_p$, I have:

$$l_p^* = l_{di}^*$$

▷ **Case (B):** $\xi \in [\bar{\xi}_p, \bar{\xi}]$**.** I start by studying one transition: Interior to interior solution. That is:

$$(1) \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = l_p^{**}$$

Let me write now the ratio between the new equilibrium and the old equilibrium. Since any function $f : x \mapsto x^{\frac{1}{\beta}}$ is positive in the domain $x \in [0,1]$ and since I am in the subset $\xi \geq \bar{\xi}_p$, I have:

$$\frac{l_p^{**}}{l_{di}^{**}} = \left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}} > 1 \quad \forall \epsilon_p < \epsilon \tag{161}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{di, p\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l_{di}^{**} < \bar{l} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = \bar{l} \\ &(3) \quad \bar{l} < l_{di}^{**} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = \bar{l} \quad \text{and} \quad l_p^* = \bar{l} \end{aligned} \tag{162}$$

Consider Scenario (2): $l_{di}^* = l_{di}^{**}$ and $l_p^* = \bar{l}$. Since $l_p^* = \min\{l_p^{**}, \bar{l}\}$ and given condition (164), then it must be the case that

$$\frac{\bar{l}}{l_{di}^{**}} = \frac{\beta}{\beta-1}\left(\frac{(1-\xi)\tau}{(\tau-1)(1+(\beta-1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi \in [\bar{\xi}_p, \bar{\xi}], \quad \forall \epsilon_p < \epsilon \tag{163}$$

In Scenario (3) the optimal coupon does not change: $l_{di}^* = \bar{l}$ and $l_p^* = \bar{l}$.

▷ **Case (C):** $\xi > \bar{\xi}$**.** I start by studying one transition: Interior to interior solution. That is:

$$(1) \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = l_p^{**}$$

which leads to

$$\frac{l_p^{**}}{l_{di}^{**}} = \left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} > 1 \quad \forall \epsilon_p < \epsilon \tag{164}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{di, p\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l_{di}^{**} < \bar{l} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = \bar{l} \\ &(3) \quad \bar{l} < l_{di}^{**} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = \bar{l} \quad \text{and} \quad l_p^* = \bar{l} \end{aligned} \tag{165}$$

Consider Scenario (2): $l_{di}^* = l_{di}^{**}$ and $l_p^* = \bar{l}$. Since $l_p^* = \min\{l_p^{**}, \bar{l}\}$ and given condition (164), then it must be the case that

$$\frac{\bar{l}}{l_{di}^{**}} = \frac{\beta}{\beta-1}\left(\frac{(1-\xi)\tau}{(\tau-1)(1+(\beta-1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi \in [\bar{\xi}_p, \bar{\xi}], \quad \forall \epsilon_p < \epsilon \tag{166}$$

In Scenario (3) the optimal coupon does not change: $l_{di}^* = \bar{l}$ and $l_p^* = \bar{l}$.

Hence, for all scenarios the equilibrium post-PSA total liability coupon is always equal or greater than the equilibrium total liability coupon chosen before this tool being introduced. □

Proof of Proposition 9. Restrict to $\xi < \overline{\xi}_p$. By comparing conditions (68) and (69) in Proposition 7 with conditions (47) and (48) in Proposition 3, I conclude the combination of deposit coupons and non-deposit coupons stays unchanged:

$$c^*_{di} = c^*_p = 0$$
$$d^*_{di} = l^*_{di} \qquad \text{and} \qquad d^*_p = l^*_p$$

The leverage increase is absorbed by deposits and the weak bank will maintain the debt structure.

Restrict to $\xi > \overline{\xi}$. By comparing conditions (68) and (69) in Proposition 7 with conditions (47) and (48) in Proposition 3, I conclude that the range of deposit and non-deposit liabilities changes due to the increase in the total leverage coupon. However, the bank can choose any combination of deposits and non-deposit liabilities such that the existing mix stays unchanged.

Restrict to $\xi \in [\overline{\xi}_p, \overline{\xi}]$. In this case, I have:

$$c^*_{di} = 0 \qquad \text{and} \qquad d^*_{di} = l^*_{di}$$
$$c^*_p = c^{**}_p \qquad \text{and} \qquad d^*_p = l^*_p$$

with

$$c^{**}_p \in \begin{cases} \left]0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right[& , \quad \text{for} \quad l^{**}_p \geq \bar{l}_p \\ \left]0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right[& , \quad \text{for} \quad l^{**}_p < \bar{l}_p \end{cases} \tag{167}$$

The domain of c^{**}_p is an open set as the Non-Negative Constraints do not bind in the optimisation problem. Hence, the optimal solution imposes the presence of both types of debt for $\xi > \overline{\xi}_p$, i.e. $d^*_p > 0$. As a result the existing debt structure with 100% of deposits will have to change, after the PSA, to include non-deposit liabilities as well. ☐

Proof of Proposition 10. Recall that $\overline{\xi}_p > \overline{\xi}$, i.e. the introduction of the PSA decreases the threshold that separates the strong from weak banks. Hence, I define three partitions: In Case (A), the bank is positioned in $\xi \in [0, \overline{\xi}_p)$, i.e. this is a "very weak bank"; In Case (B), the bank is in $\xi \in [\overline{\xi}_p, \overline{\xi})$ which means that the bank that was weak before the introduction of the PSA and now became strong. I call this the "intermediate bank". Finally, in Case (C), the bank is in $\xi \in [\overline{\xi}, 1]$ which means that this is a "strong bank" and the introduction of the PSA will not affect the structure of the bank's value function, even though it will affect the payoffs.

I may need to combine these partitions with the feasible equilibria transitions. Given that $\bar{l}$ is an upper-bound for interior solutions l^{**}_{di} or l^{**}_p, I have three possible equilibria as seen in Proposition (8):

$$\begin{array}{llll} (1) & l^{**}_{di} < l^{**}_p < \bar{l} & \Rightarrow & l^*_{di} = l^{**}_{di} \text{ and } l^*_p = l^{**}_p \\ (2) & l^{**}_{di} < \bar{l} < l^{**}_p & \Rightarrow & l^*_{di} = l^{**}_{di} \text{ and } l^*_p = \bar{l} \\ (3) & \bar{l} < l^{**}_{di} < l^{**}_p & \Rightarrow & l^*_{di} = \bar{l} \text{ and } l^*_p = \bar{l} \end{array} \tag{168}$$

To study the changes in capital structure, I must understand first the changes in the equity value claim and in the bank's total value. Let me represent the absolute change in the claims with a Δ. The change in equilibrium bank value after the introduction of private sector acquisition tool is

$$\Delta B_{di,p}(x) = B_p(x|\mathcal{E}_p) - B_{di}(x|\mathcal{E}_{di})$$

This is the change in bank's value as a consequence of a regulatory change ($B_{di} \to B_p$) and a simultaneous change in the capital structure towards the new equilibria ($\mathcal{E}_{di} \to \mathcal{E}_p$).

The changes in the claims value function from the costless deposit insurance case to the private sector acquisition case are as follows: the change in equity value is $\Delta E_{di,p}(x)$, the change in non-deposit liabilities is $\Delta C_{di,p}(x)$ and the change in deposits is $\Delta D_{di,p}$. Moreover, note that

$$\Delta B_{di,p}(x) = \Delta E_{di,p}(x) + \Delta C_{di,p}(x) + \Delta D_{di,p}$$

Finally, the capital structure ratio change is:

$$\begin{aligned} \Delta CS_{di,p}(x) &= \frac{E_p(x|\mathcal{E}_p)}{B_p(x|\mathcal{E}_p)} - \frac{E_{di}(x|\mathcal{E}_{di})}{B_{di}(x|\mathcal{E}_{di})} = \frac{E_p(x|\mathcal{E}_p)}{E_{di}(x|\mathcal{E}_{di})} - \frac{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)}{C_{di}(x|\mathcal{E}_{di}) + D_{di}(\mathcal{E}_{di})} \\ &= \frac{E_p(x|\mathcal{E}_p)}{E_{di}(x|\mathcal{E}_{di})} - \frac{L_p(\mathcal{E}_p)}{L_{di}(\mathcal{E}_{di})} \end{aligned} \tag{169}$$

And let me set the following remark useful for the rest of the proof:

Remark 6. *A function $f : x \mapsto x^{\frac{1}{\beta}}$ defined in the domain $x \in [0, 1]$ is always greater than 1.*

▷ **Case (A):** $\xi \in [0, \bar{\xi}_p)$ **.** The bank's value function and payoff structure never changes. The bank explores deposit insurance by only having deposits in its debt structure. So, closure costs are transferred to the insurance fund and become irrelevant for the bank's value function. As a result, the decrease in closure costs introduced with the PSA do not affect the weak bank. The equilibrium total leverage coupon (which equals the equilibrium deposit coupon) will not change, i.e. $l_p^* = l_{di}^*$ as discussed in Proposition (8). In conclusion, I have:

$$\Delta B_{di,p}^{(A)}(x) = 0 \qquad \wedge \qquad \Delta E_{di,p}^{(A)}(x) = 0 \qquad \Rightarrow \qquad \Delta CS_{di,p}^{(A)}(x) = 0$$

▷ **Case (B):** $\xi \in [\bar{\xi}_p, \bar{\xi})$**.** A bank deemed weak prior to the introduction of the PSA is now cosidered strong. So, from Proposition (3) and Proposition (7), the relevant equilibrium optimal total leverage coupons are:

$$\begin{aligned} l_{di}^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}\left(\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}} \\ l_p^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon_p)^{\frac{1}{\beta}} \\ \bar{l} &= \frac{(1-\xi)\rho x_0}{\rho-\mu} \end{aligned} \tag{170}$$

The difference equations for equity and total value are, respectively:

$$\Delta E_{di,p}^{(B)}(x) = \frac{1-\tau}{\rho}\left(l_{di}^* - l_p^*\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^*)^{1-\beta} - (1+(\beta-1)\xi)(l_{di}^*)^{1-\beta}\right) \tag{171}$$

$$\Delta B_{di,p}^{(B)}(x) = \frac{\tau}{\rho}\left(l_p^* - l_{di}^*\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^*)^{1-\beta} - (1+(\beta-1)\xi)(l_{di}^*)^{1-\beta}\right) \tag{172}$$

Subcase (B1): $l_{di}^* = l_{di}^{**}$ **and** $l_p^* = l_p^{**}$**.**

$$\begin{aligned} \Delta E_{di,p}^{(B1)}(x) &= \frac{1-\tau}{\rho}\left(l_{di}^{**} - l_p^{**}\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^{**})^{1-\beta} - (1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right) \\ &= \underbrace{\frac{1-\tau}{\rho}\left(1-\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}\right)l_{di}^{**}}_{<0} + \\ &+ \underbrace{\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}} - 1\right)}_{>0} \end{aligned} \tag{173}$$

I conjecture that expression (173) is negative:

$$\Delta E_{di,p}^{(B1)}(x) < 0 \Rightarrow 1 - \beta x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta} l_{di}^{**} > 0$$

and confirm it since, given the domains of the parameters, that is always the case.

Recall condition (161) from Proposition (8):

$$\begin{aligned} \Delta B_{di,p}^{(B1)}(x) &= \frac{\tau}{\rho}\left(l_p^{**} - l_{di}^{**}\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^{**})^{1-\beta} - (1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right) \\ &= \underbrace{\left(\frac{\tau}{\rho}l_{di}^{**} + \frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\right)}_{>0}\underbrace{\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{-\frac{1}{\beta}} - 1\right)}_{>0} < 0 \end{aligned} \tag{174}$$

The change in the bank's value is always positive. I can conclude that in Case (B1), the capital structure will always decrease, i.e.

$$\Delta CS^{(B1)}_{di,p}(x) < 0 \tag{175}$$

Subcase (B2): $l^*_{di} = l^{**}_{di}$ and $l^*_p = \bar{l}$. Recall functions $E^{(B)}_p(x)$ and $B^{(B)}_p(x)$ and notice that, in this subcase, the optimal total coupon after the introduction of the PSA is restricted to domain $\bar{l} \in [l^{**}_{di}, l^{**}_p]$ by Proposition (8). I study the bounds of the domain of $\bar{l}$ to show that equity value function has a root that yields no change in equity value.

Fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the new optimal total coupon level binds at the lower bound of the domain, i.e. $\bar{l} = l^{**}_{di}$. I have in this circumstance

$$\Delta E^{(B2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = \underbrace{\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l^{**}_{di})^{1-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} > 0 \tag{176}$$

and

$$\Delta B^{(B2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = \underbrace{\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l^{**}_{di})^{1-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} > 0 \tag{177}$$

Since $\bar{l} = l^{**}_{di}$, I have $\Delta L^{(B2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = 0$. Then, by condition (169) and given $\xi > \bar{\xi}_p$, the change in the capital structure is always positive:

$$\Delta CS^{(B2)}_{di,p}(x|\bar{l} = l^{**}_{di}) > 0 \tag{178}$$

Now fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the upper bound of the domain, i.e. $\bar{l} = l^{**}_p$. Making use of condition (161) from Proposition (8) and Remark (6), I get:

$$\begin{aligned}\Delta E^{(B2)}_{di,p}(x|\bar{l} = l^{**}_p) &= \frac{1-\tau}{\rho}\left(l^{**}_{di} - l^{**}_p\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l^{**}_p)^{1-\beta} - (1+(\beta-1)\xi)(l^{**}_{di})^{1-\beta}\right)\\ &= \underbrace{\frac{1-\tau}{\rho}\left(1-\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}\right)l^{**}_{di}}_{<0} + \\ &\quad + \underbrace{\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l^{**}_{di})^{1-\beta}\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}} - 1\right)}_{>0}\end{aligned} \tag{179}$$

I conjecture that expression (179) is negative:

$$\Delta E^{(B2)}_{di,p}(x|\bar{l} = l^{**}_p) < 0 \Rightarrow 1 - \beta x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta} l^{**}_{di} > 0$$

and confirm it since, given the domains of the parameters, that is always the case.

Recall condition (161) from Proposition (8):

$$\begin{aligned}\Delta B^{(B2)}_{di,p}(x|\bar{l} = l^{**}_p) &= \frac{\tau}{\rho}\left(l^{**}_p - l^{**}_{di}\right) + \frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l^{**}_p)^{1-\beta} - (1+(\beta-1)\xi)(l^{**}_{di})^{1-\beta}\right)\\ &= \underbrace{\left(\frac{\tau}{\rho}l^{**}_{di} + \frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l^{**}_{di})^{1-\beta}\right)}_{>0}\underbrace{\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{-\frac{1}{\beta}} - 1\right)}_{>0} < 0\end{aligned} \tag{180}$$

The change in the bank's value is always positive. In this case, if the bank's optimal leverage binds at the rationality constraint and the rationality constraint is equal (or sufficiently close) to the optimal unconstrained total coupon level l^{**}_p, then the capital structure will always decrease, i.e.

$$\Delta CS^{(B2)}_{di,p}(x|\bar{l} = l^{**}_p) < 0 \tag{181}$$

I conclude that, for financial institutions located in the region $\xi \in [\bar{\xi}_p, \bar{\xi}]$, i.e. that went from weak to strong with the PSA's introduction, and are forced to bind at the the rationality constraint can observe an increase or a decrease in the optimal capital structure. The change will be positive if the total coupon ($\bar{l}$) is closer to the DIS equilibrium total coupon (l_{di}^{**}) or negative if the total coupon ($\bar{l}$) is closer to the unconstrained equilibrium total coupon in the PSA case (l_p^{**})

Subcase (B3): $l_{di}^* = l_p^* = \bar{l}$. I now consider that both optimal solutions, before and after PSA's introduction, bind at the rationality constraint. The difference equations for equity and total value are, respectively:

$$\Delta E_{di,p}^{(B3)}(x) = \underbrace{\frac{(1-\tau)(\bar{l})^{1-\beta}}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} \tag{182}$$

$$\Delta B_{di,p}^{(B3)}(x) = \underbrace{\frac{(1-\tau)(\bar{l})^{1-\beta}}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} \tag{183}$$

Since $l_{di}^* = l_p^* = \bar{l}$, I have $\Delta L_{di,p}^{(B3)}(x) = 0$. Then, by condition (169) and given $\xi > \bar{\xi}_p$, the change in the capital structure is always positive:

$$\Delta CS_{di,p}^{(B3)}(x) > 0 \tag{184}$$

▷ **Case (C):** $\xi \in [\bar{\xi}, 1]$ **.** In this case, the bank is a strong bank before and after the introduction of the PSA. The bank's value function does not change as in the intermediate case (weak banks become strong banks), but the payoff structure changes and impacts the capital structure. In essence, the decrease in closure costs introduced with the PSA allows for an increase in leverage by the existing trade-off between the tax shield of debt and the closure costs. So, from Proposition (3) and Proposition (7), the relevant equilibrium optimal total leverage coupons are:

$$\begin{aligned} l_{di}^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon)^{\frac{1}{\beta}} \\ l_p^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon_p)^{\frac{1}{\beta}} \\ \bar{l} &= \frac{(1-\xi)\rho x_0}{\rho-\mu} \end{aligned} \tag{185}$$

The difference functions are in this case:

$$\Delta E_{di,p}^{(C)}(x) = \frac{1-\tau}{\rho}\left(l_{di}^* - l_p^*\right) - \frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p(l_p^*)^{1-\beta} - \epsilon(l_{di}^*)^{1-\beta}\right) \tag{186}$$

$$\Delta B_{di,p}^{(C)}(x) = \frac{\tau}{\rho}\left(l_p^* - l_{di}^*\right) - \frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p(l_p^*)^{1-\beta} - \epsilon(l_{di}^*)^{1-\beta}\right) \tag{187}$$

Subcase (C1): $l_{di}^* = l_{di}^{**}$ **and** $l_p^* = l_p^{**}$. Replacing with the equilibrium solutions for the total coupons in the difference functions, I have:

$$\Delta E_{di,p}^{(C1)}(x) = \frac{1-\tau}{\rho}\left(1 - \frac{\beta}{\beta-1}\left(\frac{x}{\frac{\beta(\rho-\mu)}{(\beta-1)(1-\xi)\rho}}\right)^{\beta}\frac{\epsilon(l_{di}^{**})^{-\beta}}{(1-\xi)}\right)\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} - 1\right)l_{di}^{**} \tag{188}$$

Now, I postulate that the difference in the equity's value function (188) is negative:

$$\Delta E_{di,p}^{(C1)}(x) < 0 \Rightarrow \left(\frac{x}{x_0}\right)^{\beta}\frac{\tau}{(\beta-1)(1-\tau)} - 1 < 0$$

which always hold for the domains of the parameters.

$$\Delta B_{di,p}^{(C1)}(x) = \left(\frac{\tau}{\rho} - \frac{1-\tau}{\rho}\frac{\beta}{\beta-1}\left(\frac{x}{\frac{\beta(\rho-\mu)}{(\beta-1)(1-\xi)\rho}}\right)^{\beta}\frac{\epsilon(l_{di}^{**})^{-\beta}}{(1-\xi)}\right)\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} - 1\right)l_{di}^{**} \tag{189}$$

I conjecture that the difference in the bank's value function (189) is positive:

$$\Delta B_{di,p}^{(C1)}(x) < 0 \Rightarrow 1 + \frac{1}{\beta-1}\left(\frac{x}{x_0}\right)^{\beta} > 0$$

which always hold for the parameters' domains. Then, by condition (169) and given $\xi > \bar{\xi}_p$, the change in the capital structure is always negative:

$$\Delta CS^{(C1)}_{di,p}(x) < 0 \tag{190}$$

Subcase (C2): $l^*_{di} = l^{**}_{di}$ **and** $l^*_p = \bar{l}$. Recall functions $E^{(B)}_p(x)$ and $B^{(B)}_p(x)$ and notice that, in this subcase, the optimal total coupon after the introduction of the PSA is restricted to domain $\bar{l} \in [l^{**}_{di}, l^{**}_p]$ by Proposition (8). I study the bounds of the domain of $\bar{l}$ to show that equity value function has a root that yields no change in equity value.

Subcase (C2.1): Lower bound $\bar{l} = l^{**}_{di}$. Fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the new optimal total coupon level binds at the lower bound of the domain, i.e. $\bar{l} = l^{**}_{di}$. I have, in this circumstance, a positive change in the equity claim value because, as assumed, $\epsilon_p < \epsilon$. Specifically:

$$\Delta E^{(C2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = -\frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p - \epsilon)\,(l^{**}_{di})^{1-\beta} > 0 \tag{191}$$

Similarly, the Bank's Total Value Function observes an increase with the introduction of the PSA:

$$\Delta B^{(C2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = -\frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p - \epsilon)\,(l^{**}_{di})^{1-\beta} > 0 \tag{192}$$

In conclusion, considering the lower bound of the domain, i.e $\bar{l} = l^{**}_{di}$. Since I have $\Delta L^{(C2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = 0$, by condition (169) and given $\xi > \bar{\xi}$, I find a positive change in capital structure:

$$\Delta CS^{(C2)}_{di,p}(x|\bar{l} = l^{**}_{di}) > 0 \tag{193}$$

Subcase (C2.2): Lower bound $\bar{l} = l^{**}_p$. Now fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the upper bound of the domain, i.e. $\bar{l} = l^{**}_p$. Making use of condition (161) from Proposition (8) and Remark (6), I get:

$$\Delta E^{(C2)}_{di,p}(x|\bar{l} = l^{**}_p) = \frac{\tau}{\rho}\left(1 - \left(\frac{\epsilon_p}{\epsilon}\right)^\beta\right)l^{**}_{di} - \frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^\beta - 1\right)\epsilon(l^{**}_{di})^{1-\beta} \tag{194}$$

I postulate that this equation is negative and find that:

$$\Delta E^{(C2)}_{di,p}(x) < 0 \Rightarrow \left(\frac{x}{x_0}\right)^\beta \frac{\tau}{(\beta-1)(1-\tau)} - 1 < 0$$

which always hold for the domains of the parameters. When the corner solution $\bar{l}$ is sufficiently closer to the interior solution l^{**}_p, the equity claim suffers a decrease in value. That occurs because the value increase of facing lower bankruptcy costs (with the PSA) is superseded by the increase in leverage which decreases the cash-flows available for shareholders and increase the intervention trigger, .i.e. the regulator is expected to close the bank sooner than in the previous case.

$$\Delta B^{(C2)}_{di,p}(x|\bar{l} = l^{**}_p) = \frac{\tau}{\rho}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^\beta - 1\right)l^{**}_{di} - \frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^\beta - 1\right)\epsilon(l^{**}_{di})^{1-\beta} \tag{195}$$

I need to confirm if the first term (positive) can always cover the second term (negative). Hence, I conjecture that the difference in the bank's value function (195) is positive:

$$\Delta B^{(C2)}_{di,p}(x|\bar{l} = l^{**}_p) > 0 \Rightarrow 1 + \frac{1}{\beta-1}\left(\frac{x}{x_0}\right)^\beta > 0$$

This last expression always hold for the parameters' domains. As a result, by condition (169) and given $\xi > \bar{\xi}$, the change in the capital structure is always negative:

$$\Delta CS^{(C2)}_{di,p}(x|\bar{l} = l^{**}_p) < 0 \tag{196}$$

Threshold leverage coupon: $\bar{l} = \tilde{l}$. If the optimal leverage coupon $\bar{l}$ changes signs within its domain, by Bolzano's Theorem, there exists a level of total leverage coupon that generates no change in the equity claim's value. I let that level be represented by a tilde, i.e. $\tilde{l}$. With a few standard calculations, I obtain:

$$\Delta E^{(C2)}_{di,p}(x|\bar{l} = \tilde{l}) = 0 \Rightarrow \tilde{l}^{(C2)} = \left(\frac{\frac{\rho}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p - \epsilon\left(\left(\frac{\tau}{\tau-1}\frac{1-\xi}{\beta\epsilon}\right)^{-\frac{1}{\beta}}\frac{\beta-1}{\beta}\right)^{1-\beta}\right)}{\left(\frac{\tau}{\tau-1}\frac{1-\xi}{\beta\epsilon}\right)^{-\frac{1}{\beta}}\frac{\beta-1}{\beta}}\right)^{\frac{1}{\beta}} \tag{197}$$

Subcase (C3): $l^*_{di} = \bar{l}$ **and** $l^*_p = \bar{l}$. In this subcase, the bank maintains the corner solution, binding at the rationality constraint maximum leverage coupon. Since $\epsilon_p < \epsilon$ I have, respectively, for equity and total value:

$$\Delta E^{(C3)}_{di,p}(x) = -\frac{1-\tau}{\rho-\mu}x^\beta \left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta} (\epsilon_p - \epsilon)\,(\bar{l})^{1-\beta} > 0 \tag{198}$$

$$\Delta B^{(C3)}_{di,p}(x) = -\frac{1-\tau}{\rho-\mu}x^\beta \left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta} (\epsilon_p - \epsilon)\,(\bar{l})^{1-\beta} \tag{199}$$

In conclusion, considering the lower bound of the domain, i.e $\bar{l} = l^{**}_{di}$. Since I have $\Delta L^{(C3)}_{di,p}(x) = 0$, by condition (169) and given $\xi > \bar{\xi}$, I find a positive change in capital structure:

$$\Delta CS^{(C3)}_{di,p}(x) > 0 \tag{200}$$

□

F Appendix of Bail-in Case

Proof of Lemma 5. At the bail-in trigger x^1_{bi}, the participation constraint for shareholders is:

$$(1-\theta)\left(E_{2,1}(x^1_{bi}) + C_{2,1}(x^1_{bi})\right) \geq \max\left\{(1-\tau)\left[(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\} \tag{201}$$

and for non-deposit creditors is:

$$\theta\left(E_{2,1}(x^1_{bi}) + C_{2,1}(x^1_{bi})\right) \geq \min\left\{(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\} \tag{202}$$

I get condition (87) by solving these inequalities for θ and restricting the boundaries to $\theta \in [0,1]$.

□

Proof of Proposition 11. After the regulatory changes, the optimisation problem for Scenario 2 is defined as:

$$
\begin{aligned}
(c^*_{bi}, d^*_{bi}) \quad & \in \underset{\{c_{bi}, d_{bi}\}}{\arg\max}\; B_{bi}(x_0) & \\
\text{s.t.} \quad & x^1_{bi} := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} & \text{[Bail-In Trigger]} \\
& x^2_{bi} := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} & \text{[Closure Trigger]} \\
& c_{bi} + d_{bi} \leq \frac{\rho(1-\xi)}{\rho-\mu}x_0 & \text{[Initial Capital Requirement]} \\
& c_{bi} \geq 0 \quad , \quad d_{bi} \geq 0 & \text{[Non-Negative Coupons]}
\end{aligned}
$$

▷ **Value Function.** I must restrict the problem to the feasible cases. First notice that I have two major cases: Case A when $\xi < \bar{\xi}_p$ and Case B when $\xi \geq \bar{\xi}_p$. In this Scenario the Bank's Value Function will then be:

$$B_{bi}(x_0) = \begin{cases} (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^\beta \frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^\beta \left(\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) & , \quad \xi < \bar{\xi}_p \qquad \text{[Case A]} \\ (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^\beta \frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^\beta \left(\epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right) & , \quad \xi \geq \bar{\xi}_p \qquad \text{[Case B]} \end{cases}$$

▷ **Constraint Set Feasibility.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_{bi} = \left\{(c_{bi}, d_{bi}) \in \mathbb{R}^2 | g(c_{bi}, d_{bi}) \leq 0\right\}$$

with

$$g(c_{bi}, d_{bi}) = \begin{pmatrix} g_1(c_{bi}, d_{bi}) \\ g_2(c_{bi}) \\ g_3(d_{bi}) \end{pmatrix} = \begin{pmatrix} c_{bi} + d_{bi} - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -c_{bi} \\ -d_{bi} \end{pmatrix}$$

Let $Q_{bi} := \frac{(1-\xi)\rho}{\rho-\mu}x_0$ be a family of constants and let (ρ, μ, x_0, ξ) be such that $Q_{bi} > 0$. Then, I secure the feasible set $\mathcal{G}_{bi}$ is compact which, by the Weierstrass Theorem, guarantees the existence of a maximising solution.

▷ **Constraint Qualification.** Consider the Constraint Qualification matrix

$$CQ := \nabla g(c_{bi}, d_{bi}) = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{203}$$

I immediately see that, for any possible combinations of binding constraints, $Rank(CQ) \in \{1, 2\}$. Therefore, I can exclude the case where three constraints are binding.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers χ_1 for the NBR Condition and χ_2 for the Rationality Constraint. For Case A:

$$\begin{aligned}\mathscr{L}(c_{bi}, d_{bi})|_{\xi<\bar{\xi}_p} =&(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^\beta \frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^\beta \left(\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) \\ &+ \chi_1\left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{bi}+d_{bi})\right) + \chi_2 c_{bi} + \chi_3 d_{bi}\end{aligned} \tag{204}$$

And for Case B:

$$\begin{aligned}\mathscr{L}(c_{bi}, d_{bi})|_{\xi\geq\bar{\xi}_p} =&(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^\beta \frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^\beta \left(\epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right) \\ &+ \chi_1\left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{bi}+d_{bi})\right) + \chi_2 c_{bi} + \chi_3 d_{bi}\end{aligned} \tag{205}$$

▷ **Kuhn-Tucker Conditions.** For both cases, the complementary slack conditions are:

$$\chi_1\left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{bi}+d_{bi})\right) = 0 \tag{206}$$

$$\chi_2 c_{bi} = 0 \tag{207}$$

$$\chi_3 d_{bi} = 0 \tag{208}$$

and Kuhn-Tucker multiplier constraints are:

$$\chi_1 \geq 0 \quad , \qquad \chi_2 \geq 0 \quad , \qquad \chi_3 \geq 0 \tag{209}$$

Only solutions satisfying (206)-(209) are admissible.

▷ **Corner Solutions.** I have six possible candidates to admissible corner solutions. To distinguish them I use, just for the sake of exposure in this proof, the superscript (j). Here, $j \in \{1, ..., 6\}$ is the solution candidate under study. For example, in the first solution candidate, I need to find $c^{(1)}_{bi}$, $d^{(1)}_{bi}$ and $\chi^{(1)}_1$.

1. $g_1(c_{bi}, d_{bi})$ is binding, i.e. $(\chi_1 > 0, \chi_2 = \chi_3 = 0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c^{(1)}_{bi}, d^{(1)}_{bi})$ and the Kuhn-Tucker multiplier $\chi^{(1)}_1$:

$$c^{(1)}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)x_0 & , \quad \xi < \bar{\xi}_p \qquad \text{[Case A]} \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & , \quad \xi \geq \bar{\xi}_p \qquad \text{[Case B]} \end{cases}$$

$$d^{(1)}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi < \bar{\xi}_p \qquad \text{[Case A]} \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi \geq \bar{\xi}_p \qquad \text{[Case B]} \end{cases}$$

$$\chi^{(1)}_1 = \begin{cases} \tau + \tau\left(\frac{\beta-1}{\beta}\right)^\beta\left(\beta-1-\beta\left(\frac{\tau-1}{\tau}\frac{((\beta-1)\xi+1)}{1-\xi}\right)^{1/\beta}\right) & , \quad \xi < \bar{\xi}_p \qquad \text{[Case A]} \\ \tau + \tau\left(\frac{\beta-1}{\beta}\right)^\beta\left(\beta-1-\beta\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right) & , \quad \xi \geq \bar{\xi}_p \qquad \text{[Case B]} \end{cases}$$

Solution is admissible if and only $\chi^{(1)}_1 > 0$, which is guaranteed when:

$$\left(\frac{\tau-1}{\tau}\frac{((\beta-1)\xi+1)}{1-\xi}\right)^{1/\beta} > \frac{\beta-1}{\beta}, \quad \text{for} \quad \xi < \bar{\xi}_{bi} \qquad \text{[Case A]}$$

$$\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} > \frac{\beta-1}{\beta}, \quad \text{for} \quad \xi \geq \bar{\xi}_{bi} \qquad \text{[Case B]}$$

If these conditions are guaranteed, the solution candidate 1 is admissible as a valid solution.

2. $g_2(c_{bi})$ is binding, i.e. $(\chi_1 = 0, \chi_2 > 0, \chi_3 = 0)$ In this solution candidate, the Non-Negative Constraint for Non-Deposit Liabilities is the only constraint binding. I solve for the optimal coupons $(c_{bi}^{(2)}, d_{bi}^{(2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(2)}$:

$$c_{bi}^{(2)} = 0$$

$$d_{bi}^{(2)} = \begin{cases} \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi < \bar{\xi}_p \quad \text{[Case A]} \\ \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi \geq \bar{\xi}_p \quad \text{[Case B]} \end{cases}$$

$$\chi_2^{(2)} = \begin{cases} \frac{\tau}{\rho}\left(\frac{\tau(1-\xi)}{(\tau-1)((\beta-1)\xi+1)} - 1\right) & , \quad \xi < \bar{\xi}_p \quad \text{[Case A]} \\ \frac{\tau}{\rho}\left(\frac{\tau(1-\xi)}{\epsilon_p\beta(\tau-1)} - 1\right) & , \quad \xi \geq \bar{\xi}_p \quad \text{[Case B]} \end{cases}$$

Let me restrict to $\xi > \frac{1}{1-\beta}$ (as I will see, this is a useful restriction for the unconstrained problem as well). A solution is admissible if and only $\chi_2^{(2)} > 0$, which is guaranteed when:

$$\xi < \frac{1}{\beta(\tau-1)+1}, \quad \text{for} \quad \xi < \bar{\xi}_p \quad \text{[Case A]}$$

$$\xi < 1 - \frac{\tau-1}{\tau}\epsilon_p\beta, \quad \text{for} \quad \xi \geq \bar{\xi}_p \quad \text{[Case B]}$$

Then, the solution candidate 2 is a valid solution.

3. $g_3(d_{bi})$ is binding, i.e. $(\chi_1 = 0, \chi_2 = 0, \chi_3 > 0)$ In this solution, the Non-Negative Constraint for Deposits is the only constraint binding. I need to solve for the optimal coupons $(c_{bi}^{(3)}, d_{bi}^{(3)})$ and the Kuhn-Tucker multiplier $\chi_3^{(3)}$. However, I find that, for $\xi \in [0, 1]$:

$$\chi_2^{(3)} = \frac{\tau}{(\beta-1)\rho} < 0$$

This candidate solution is never an admissible solution.

4. $g_1(c_{bi}, d_{bi})$ and $g_2(c_{bi})$ bind, i.e. $(\chi_1 > 0, \chi_2 > 0, \chi_3 = 0)$ In this solution, the Rationality Constraint and the Non-Negativity Constraint for Non-Deposit Liabilities are both binding. I solve for the optimal coupons $(c_{bi}^{(4)}, d_{bi}^{(4)})$ and the Kuhn-Tucker multipliers $\chi_1^{(4)}$ and $\chi_2^{(4)}$:

$$c_{bi}^{(4)} = 0$$

$$d_{bi}^{(4)} = \frac{\rho(1-\xi)}{\rho-\mu} x_0$$

$$\chi_1^{(4)} = \begin{cases} \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{1-\tau}{1-\xi}((\beta-1)\xi+1)+\tau & , \quad \xi < \bar{\xi}_p \quad \text{[Case A]} \\ \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{1-\tau}{1-\xi}\epsilon_p\beta+\tau & , \quad \xi \geq \bar{\xi}_p \quad \text{[Case B]} \end{cases}$$

$$\chi_2^{(4)} = \begin{cases} \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{\beta\xi(\tau-1)+\xi-1}{(\xi-1)\rho} & , \quad \xi < \bar{\xi}_p \quad \text{[Case A]} \\ \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{\epsilon_p\beta(\tau-1)+(\xi-1)\tau}{(\xi-1)\rho} & , \quad \xi \geq \bar{\xi}_p \quad \text{[Case B]} \end{cases}$$

The solution candidate is admissible if and only if $\chi_1^{(4)} > 0$ and $\chi_2^{(4)} > 0$ for any acceptable range of parameters.

5. $g_1(c_{bi}, d_{bi})$ and $g_3(d_{bi})$ bind, i.e. $(\chi_1 > 0, \chi_2 = 0, \chi_3 > 0)$ I check if it is possible to bind the Rationality Constraint and the Non-Negative Constraint for Deposits. I solve for the optimal coupons $(c_{bi}^{(5)}, d_{bi}^{(5)})$ and the Kuhn-Tucker multipliers $\chi_1^{(5)}$ and $\chi_3^{(5)}$ but find that:

$$\chi_3^{(5)} = -\frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}\tau}{\rho} < 0$$

The solution candidate is not admissible for $\chi_2^{(5)} < 0$.

6. $g_2(c_{bi})$ and $g_3(d_{bi})$ bind, i.e. $(\chi_1 = 0, \chi_2 > 0, \chi_3 > 0)$ I check if it is possible to bind the two Non-Negative Constraints. I solve for the optimal coupons $(c_{bi}^{(6)}, d_{bi}^{(6)})$ and the Kuhn-Tucker multipliers $\chi_2^{(6)}$ and $\chi_3^{(6)}$ but find that:

$$\chi_2^{(6)} = -\frac{\tau}{\rho} < 0$$

The solution candidate is not admissible for $\chi_2^{(6)} < 0$.

▷ **Interior Solution** $(\chi_1 = \chi_2 = \chi_3 = 0)$ **- First Order Conditions.** Fixing the Kuhn-Tucker multipliers at zero and rearranging, I obtain:

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}d_{bi}}\Big|_{\xi\in[0,\bar{\xi}_p)} = 0 \Rightarrow d_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}x_0$$
$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}d_{bi}}\Big|_{\xi\in[\bar{\xi}_p,1]} = 0 \Rightarrow d_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}x_0 \tag{210}$$

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}c_{bi}}\Big|_{\xi\in[0,\bar{\xi}_p)} = 0 \Rightarrow c_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0$$
$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}c_{bi}}\Big|_{\xi\in[\bar{\xi}_p,1]} = 0 \Rightarrow c_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 \tag{211}$$

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm the concavity of $B_{bi}(x_0)$ for the unconstrained situation. Taking second order conditions and rearranging, I guarantee d_{bi}^* and c_{bi}^* are maximising arguments for the firm value function conditional on subset $\xi \in [0, \bar{\xi}_p)$ and restricting to one of the following pairs of necessary and sufficient conditions:

$$|\beta| \geq 1 \wedge \xi < \min\left\{\frac{(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1} - \tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2}}{\tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2} + (\beta-1)(\tau-1)\left(\frac{1}{d_{bi}}\right)^{\beta+1}}, 1 + \frac{\epsilon_p\beta(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{-\beta-2}}{(\beta+1)c_{bi}\tau}\right\}$$

or, instead

$$|\beta| \leq 1 \wedge \xi > \max\left\{\frac{(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1} - \tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2}}{\tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2} + (\beta-1)(\tau-1)\left(\frac{1}{d_{bi}}\right)^{\beta+1}}, 1 + \frac{\epsilon_p\beta(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{-\beta-2}}{(\beta+1)c_{bi}\tau}\right\}$$

Then, the SOC for the firm is, for $\xi \in [0, \bar{\xi}_p)$:

$$\frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{d_{bi}}^2} = \underbrace{-\frac{\beta}{\rho}}_{>0}\left(\underbrace{\frac{1+(\beta-1)\xi}{d}\frac{1-\tau}{1-\xi}\left(\frac{x_0}{x_{bi}^2}\right)^{\beta} + \frac{\tau c_{bi}}{(c_{bi}+d_{bi})^2}(1+\beta)\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}}_{<0}\right) < 0$$

and for $\xi \in [\bar{\xi}_p, 1]$, the SOC comes as:

$$\frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{d_{bi}}^2} = \underbrace{-\frac{\beta}{\rho}}_{>0}\left(\underbrace{\frac{\epsilon_p\beta}{d_{bi}}\frac{1-\tau}{1-\xi}\left(\frac{x_0}{x_{bi}^2}\right)^{\beta} + \frac{\tau c_{bi}}{(c_{bi}+d_{bi})^2}(1+\beta)\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}}_{<0}\right) < 0$$

The SOC for c_{bi} is the same for all $\xi \in [0, 1]$:

$$\frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{c_{bi}}^2}\Big|_{\xi\in[0,\bar{\xi}_p)} = \frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{c_{bi}}^2}\Big|_{\xi\in[\bar{\xi}_p,1]} = \underbrace{\frac{\beta\tau}{\rho}}_{<0}\underbrace{\frac{2d_{bi}+(1-\beta)c_{bi}}{(c_{bi}+d_{bi})^2}}_{>0}\underbrace{\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}}_{>0} < 0$$

For negative SOCs for the sufficient stronger pair of condition that heavily simplifies my parameterisation:

$$\xi > \frac{1}{1-\beta} \quad \wedge \quad |\beta| \geq 1 \tag{212}$$

This eliminates, however, several cases where the maximisation property of controls (d_{bi}, c_{bi}) would be secured.

▷ **Equilibrium Solution.** By solving the constrained optimisation problem, I can synthesise the main equilibrium result. First, let me denominate the optimal parameters as:

$$\begin{aligned} \overline{c}_{bi} &:= c_{bi}^{(1)} & \overline{d}_{bi} &:= d_{bi}^{(1)} \\ \underline{c}_{bi} &:= c_{bi}^{(2)} & \underline{d}_{bi} &:= d_{bi}^{(2)} \\ \hat{c}_{bi} &:= c_{bi}^{(4)} & \hat{d}_{bi} &:= d_{bi}^{(4)} \end{aligned}$$

and, taking the Rationality Constraint at its binding point, let me write:

$$\bar{l}_{bi} := \frac{(1-\xi)\rho}{\rho-\mu} x_0$$

and also let me write the total periodic coupon for the interior solution as:

$$l_{bi}^{**} = c_{bi}^{**} + d_{bi}^{**}$$

Then, the equilibrium solution (c_{bi}^*, d_{bi}^*) is defined by the periodic coupon c_{bi}^* for non-deposit liabilities and by the periodic coupon d_{bi}^* for deposits as:

$$(c_{bi}^*, d_{bi}^*) = \begin{cases} (\hat{c}_{bi}, \hat{d}_{bi}) & \text{for} \quad l_{bi}^{**} \geq \bar{l}_{bi} \quad \wedge c_{bi}^{**} \leq 0 \\ (\overline{c}_{bi}, \overline{d}_{bi}) & \text{for} \quad l_{bi}^{**} \geq \bar{l}_{bi} \quad \wedge c_{bi}^{**} > 0 \\ (\underline{c}_{bi}, \underline{d}_{bi}) & \text{for} \quad l_{bi}^{**} < \bar{l}_{bi} \quad \wedge c_{bi}^{**} \leq 0 \\ (c_{bi}^{**}, d_{bi}^{**}) & \text{for} \quad l_{bi}^{**} < \bar{l}_{bi} \quad \wedge c_{bi}^{**} > 0 \end{cases} \tag{213}$$

where, when the Rationality Constraint and Non-Negative Constraint for Non-Deposit Liabilities bind, I have:

$$\hat{c}_{bi} = 0 \qquad \text{and} \qquad \hat{d}_{bi} = \frac{(1-\xi)\rho}{\rho-\mu} x_0 \tag{214}$$

when only the Rationality Constraint binds, I have:

$$\overline{c}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)x_0 & , \quad \xi < \overline{\xi}_p \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{215}$$

$$\overline{d}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi < \overline{\xi}_p \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases}$$

when only the Non-Negative Constraint for Non-Deposit Liabilities binds:

$$\underline{c}_{bi} = 0 \qquad \text{and} \qquad \underline{d}_{bi} = \begin{cases} \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi < \overline{\xi}_p \\ \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{216}$$

and, at last, the interior solution (no constraint is binding) yields:

$$d_{bi}^{**} = \begin{cases} (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{217}$$

$$c_{bi}^{**} = \begin{cases} (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases}$$

□

Proof of Proposition 12. Consider two cases concerning each type of bank: Case (A) studies weak banks ($\xi < \bar{\xi}_p$) while Case (B) studies strong banks ($\xi \geq \bar{\xi}_p$).

Remark 7. *A function $f : \kappa \mapsto \kappa^{-\frac{1}{\beta}} \left(1 - \beta \left(1 - \kappa^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}}$ with $\kappa \in \mathbb{R}_+$ and $\beta \in \mathbb{R}_-$ is always equal or greater than 1:*

$$\lim_{\kappa \downarrow 0} f(\kappa) = +\infty \qquad \lim_{\kappa \uparrow +\infty} f(\kappa) = +\infty$$
$$\lim_{\kappa \downarrow 1} f(\kappa) = 1 \qquad \lim_{\kappa \uparrow 1} f(\kappa) = 1$$

Remark 8. *I restrict to the tuple of parameters (μ, σ, ρ, ξ) that secures $\xi > \frac{1}{1-\beta}$*

▷ **Case (A):** $\xi \in [0, \bar{\xi}_p)$**.** I start by studying the interior to interior transition.That is:

$$(1) \quad l_p^* = l_p^{**} \quad \text{and} \quad l_{bi}^* = l_{bi}^{**}$$

Write the ratio between the new equilibrium total debt coupon and the old equilibrium total debt coupon:

$$\frac{l_{bi}^{**}}{l_p^{**}} = \left(\frac{\tau - 1}{\tau} \frac{\xi(\beta - 1) + 1}{1 - \xi}\right)^{-\frac{1}{\beta}} \left(1 - \beta \left(1 - \left(\frac{\tau - 1}{\tau} \frac{\xi(\beta - 1) + 1}{1 - \xi}\right)^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}} \tag{218}$$

Considering a change of variables $\kappa = \left(\frac{\tau-1}{\tau} \frac{\xi(\beta-1)+1}{1-\xi}\right)^{-\frac{1}{\beta}}$ and using the result obtained in Remark 7, I conclude that

$$\frac{l_{bi}^{**}}{l_p^{**}} \geq 1, \qquad \forall \xi < \bar{\xi}_p \tag{219}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{p, bi\}$, I can restrict the rest of the problem to two other scenarios:

$$\begin{aligned} &(2) \quad l_p^{**} < \bar{l} < l_{bi}^{**} \quad \Rightarrow \quad l_p^* = l_p^{**} \quad \text{and} \quad l_{bi}^* = \bar{l} \\ &(3) \quad \bar{l} < l_p^{**} < l_{bi}^{**} \quad \Rightarrow \quad l_p^* = \bar{l} \quad \text{and} \quad l_{bi}^* = \bar{l} \end{aligned} \tag{220}$$

Consider Scenario (2): $l_p^* = l_p^{**}$ and $l_{bi}^* = \bar{l}$. Since $l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}\}$ and given condition (219), then it must be the case that

$$\frac{\bar{l}}{l_p^{**}} = \left(1 - \beta \left(1 - \left(\frac{\tau - 1}{\tau} \frac{\xi(\beta - 1) + 1}{1 - \xi}\right)^{\frac{1}{\beta}}\right)\right)^{-\frac{1}{\beta}} \geq 1 \tag{221}$$

In Scenario (3) the optimal coupon does not change: $l_p^* = \bar{l}$ and $l_{bi}^* = \bar{l}$.

▷ **Case (B):** $\xi \in [\bar{\xi}_p, 1]$**.** I start by studying the interior to interior solution. That is:

$$(1) \quad l_p^* = l_p^{**} \quad \text{and} \quad l_{bi}^* = l_{bi}^{**}$$

I write the ratio

$$\frac{l_{bi}^{**}}{l_p^{**}} = \left(\frac{\tau - 1}{\tau} \frac{\epsilon_p \beta}{1 - \xi}\right)^{-\frac{1}{\beta}} \left(1 - \beta \left(1 - \left(\frac{\tau - 1}{\tau} \frac{\epsilon_p \beta}{1 - \xi}\right)^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}} \tag{222}$$

Considering a change of variables $\kappa = \left(\frac{\tau-1}{\tau} \frac{\epsilon_p \beta}{1-\xi}\right)^{-\frac{1}{\beta}}$ and using the result obtained in Remark 7, I conclude that

$$\frac{l_{bi}^{**}}{l_p^{**}} \geq 1, \qquad \forall \xi \geq \bar{\xi}_p \tag{223}$$

In Scenario (2): $l_p^* = l_p^{**}$ and $l_{bi}^* = \bar{l}$. Since $l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}\}$ and given condition (223), then it must be the case that

$$\frac{\bar{l}}{l_p^{**}} = \left(1 - \beta \left(1 - \left(\frac{\tau - 1}{\tau} \frac{\epsilon_p \beta}{1 - \xi}\right)^{\frac{1}{\beta}}\right)\right)^{-\frac{1}{\beta}} \geq 1 \tag{224}$$

In Scenario (3) the optimal coupon does not change: $l_p^* = \bar{l}$ and $l_{bi}^* = \bar{l}$. In summary, the total debt coupon will never decrease with the introduction of the Bail-in tool:

$$l_{bi}^* \geq l_p^* \tag{225}$$

□

Proof of Proposition 13. The debt structure ratio change is:

$$\Delta DS_{p,bi}(x) = \frac{C_{bi}(x|\mathcal{E}_{bi})}{C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi})} - \frac{C_p(x|\mathcal{E}_p)}{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)} = \frac{C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) - C_p(x|\mathcal{E}_p)D_{bi}(\mathcal{E}_{bi})}{\left(C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi})\right)\left(C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)\right)} \quad (226)$$

I can restrict the problem to study the difference

$$C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) - C_p(x|\mathcal{E}_p)D_{bi}(\mathcal{E}_{bi}) \quad (227)$$

I study the two cases concerning each type of bank: Case (A) is focused on weak banks ($\xi < \bar{\xi}_p$) while Case (B) is focused on strong banks ($\xi \geq \bar{\xi}_p$).

▹ **Case (A):** $\xi \in [0, \bar{\xi}_p)$**.** Before the bail-in is introduced, the debt structure of the weak bank is entirely constituted with deposits. In detail, $d_p^* = l_p^*$ and $c_p^* = 0$. This implies that the value function of non-deposit liabilities of weak banks is:

$$C_p(x|\mathcal{E}_p, \xi < \bar{\xi}_p) = 0$$

With the bail-in, I always have $d_{bi}^* > 0$ and $c_{bi}^* \geq 0$ which implies $D_{bi}(\mathcal{E}_{bi}) > 0$ and $C_{bi}(x|\mathcal{E}_{bi}) \geq 0$. Hence, from expression (227), I conclude:

$$C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) \geq 0 \Rightarrow \Delta DS_{p,bi}(x) \geq 0$$

▹ **Case (B):** $\xi \in [\bar{\xi}_p, 1]$**.** In the strong bank case, I can simplify expression (227) by substituting the value of non-deposit liabilities in the PSA case $C_p = c_p/\rho$, the value of deposits in the PSA case $D_p = d_p/\rho$ and the value of deposits in the Bail-in case $D_{bi} = d_{bi}/\rho$. So, I have:

$$\frac{d_p^*}{\rho}\left(C_{bi}(x|\mathcal{E}_{bi}) - \frac{c_p^* d_{bi}^*}{\rho d_p^*}\right) \quad (228)$$

The debt structure of strong banks is undefined for the PSA case. The strong bank can select any pair (c_p^*, d_p^*) provided it respects the ranges presented in Proposition 7.

Without loss of generality, I write $C_{bi}(x|\mathcal{E}_{bi}) \equiv \overline{C}^* \geq 0$ as a constant under the equilibrium capital structure in the bail-in case. Furthermore, from condition (68) I write $d_p^* = l_p^* - c_p^*$. Finally, I replace these identities in condition (228) and, use the range defined in condition (69) to take limits over $c_p^* \in (0, l_p^*)$:

$$\begin{aligned} \overline{C}^* - \lim_{c_p^* \to 0} \frac{c_p^* d_{bi}^*}{\rho(l_p^* - c_p^*)} &= \overline{C}^* \geq 0 \quad \Rightarrow \Delta DS_{p,bi}(x) \geq 0 \\ \overline{C}^* - \lim_{c_p^* \to l_p^*} \frac{c_p^* d_{bi}^*}{\rho(l_p^* - c_p^*)} &= -\infty < 0 \quad \Rightarrow \Delta DS_{p,bi}(x) < 0 \end{aligned} \quad (229)$$

Condition (229) shows that, depending on the undefined mix (c_p^*, d_p^*), a strong bank can increase, decrease or maintain its debt structure when it changes towards the unique bail-in equilibrium coupon mix (c_{bi}^*, d_{bi}^*). Therefore, the change in debt structure of strong banks is undefined when bail-ins are introduced.

□

Proof of Proposition 14. The change in equilibrium bank value after the introduction of the bail-in tool is

$$\Delta B_{p,bi}(x) = B_{bi}(x|\mathcal{E}_{bi}) - B_p(x|\mathcal{E}_p)$$

This is the change in bank's value as a consequence of a regulatory change ($B_p \to B_{bi}$) and a simultaneous change in the capital structure towards the new equilibria ($\mathcal{E}_p \to \mathcal{E}_{bi}$).

The change in equity value with the introduction of a Bail-in is $\Delta E_{p,bi}(x)$, the change in non-deposit liabilities is $\Delta C_{p,bi}(x)$ and the change in deposits is $\Delta D_{p,bi}$. Moreover,

$$\Delta B_{p,bi}(x) = \Delta E_{p,bi}(x) + \Delta C_{p,bi}(x) + \Delta D_{p,bi}(x)$$

Finally, the capital structure ratio change is:

$$\Delta CS_{p,bi}(x) = \frac{E_{bi}(x|\mathcal{E}_{bi})}{B_{bi}(x|\mathcal{E}_{bi})} - \frac{E_p(x|\mathcal{E}_p)}{B_{di}(x|\mathcal{E}_p)} = \frac{E_{bi}(x|\mathcal{E}_{bi})}{E_p(x|\mathcal{E}_p)} - \frac{C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi})}{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)} \quad (230)$$

This expression can be further simplified. Consider first the PSA case. By Proposition 7, the weak bank only uses deposits, i.e. $d_{bi}^* = l_{bi}^*$. This means that the entire credit claim is protected by the DIS and so the value of the credit claim is not a function of the the closure trigger and the state variable. If, instead, the institution is a strong bank, then it always pays creditors at closure. Hence, the total credit claim also does not change with the closure decisions. For both types of banks under the PSA, I can write:

$$C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p) = L_p(\mathcal{E}_p) = \frac{l_p^*}{\rho}, \qquad \forall \xi \in [0,1]$$

Consider now the Bail-in case. From Proposition 11, the weak bank is not restricted to 100% of deposits since, in some cases, having non-deposit liabilities is desirable. Deposits are always protected by the DIS and so and non-deposit liabilities have their claim preserved ex-ante by Assumption 88. If the bank is instead a strong bank, then it always pays creditors at closure. Hence, the total credit claim does not change with the closure decisions. Hence, I can write for both types of banks in the Bail-in case:

$$C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi}) = L_{bi}(\mathcal{E}_{bi}) = \frac{l^*_{bi}}{\rho}, \qquad \forall \xi \in [0,1]$$

I simplify condition (230):

$$\Delta CS_{p,bi}(x) = \frac{E_{bi}(x|\mathcal{E}_{bi})}{E_p(x|\mathcal{E}_p)} - \frac{l^*_{bi}}{l^*_p} = \frac{1}{E_p(x|\mathcal{E}_p)}\left(E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p}\right) \tag{231}$$

and restrict the problem to the study of expression:

$$E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} \tag{232}$$

From Proposition 12, I have three possible equilibrium transitions:

$$\begin{aligned}
&(1) \quad l^{**}_p < l^{**}_{bi} < \bar{l} &&\Rightarrow \quad l^*_p = l^{**}_p \quad \text{and} \quad l^*_{bi} = l^{**}_{bi} \\
&(2) \quad l^{**}_p < \bar{l} < l^{**}_{bi} &&\Rightarrow \quad l^*_{di} = l^{**}_{di} \quad \text{and} \quad l^*_p = \bar{l} \\
&(3) \quad \bar{l} < l^{**}_{di} < l^{**}_p &&\Rightarrow \quad l^*_{di} = \bar{l} \quad \text{and} \quad l^*_p = \bar{l}
\end{aligned} \tag{233}$$

▷ **Case (A):** $\xi \in [0, \bar{\xi}_p)$ **.** I write the parameters κ_1 and $\overline{\kappa}_1$ as

$$\kappa_1 := \frac{\tau - 1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi} \tag{234}$$

and

$$\overline{\kappa}_1 := \max\{1, \kappa_1\} \tag{235}$$

From Propositions 7 and 11, I can write the following relationships between optimal coupons in the bail-in and the PSA cases regardless of having an interior or corner solution:

$$\frac{l^*_{bi}}{l^*_p} = \left(\frac{1-\beta(1-\overline{\kappa}_1^{\frac{1}{\beta}})}{\overline{\kappa}_1}\right)^{\frac{1}{\beta}} \qquad \in [1, +\infty) \tag{236}$$

$$\frac{d^*_{bi}}{l^*_p} = \left(1-\beta(1-\overline{\kappa}_1^{\frac{1}{\beta}})\right)^{\frac{1}{\beta}} \qquad \in [0,1] \tag{237}$$

$$\frac{c^*_{bi}}{l^*_p} = \left(1-\beta(1-\overline{\kappa}_1^{\frac{1}{\beta}})\right)\left(\frac{1-\beta(1-\overline{\kappa}_1^{\frac{1}{\beta}})}{\overline{\kappa}_1}\right)^{\frac{1}{\beta}} \qquad \in [0, +\infty) \tag{238}$$

$$\frac{d^*_{bi}}{l^*_{bi}} = \overline{\kappa}_1^{\frac{1}{\beta}} \qquad \in [0,1] \tag{239}$$

$$\frac{c^*_{bi}}{l^*_{bi}} = 1 - \overline{\kappa}_1^{\frac{1}{\beta}} \qquad \in [0,1] \tag{240}$$

Condition (232) in the weak banks' case is:

$$\begin{aligned}
E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} =& \frac{1-\tau}{\rho-\mu}x_0\left(1-\frac{l^*_{bi}}{l^*_p}\right) - \left(\frac{x}{x^1_{bi}}\right)^{\beta}\tau\frac{c_{bi}}{\rho} \\
&- \left(\frac{x}{x^2_{bi}}\right)^{\beta}(1-\tau)\left(\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) + \left(\frac{x}{x_p}\right)^{\beta}(1-\tau)\left(\frac{x^1_{bi}}{\rho-\mu} - \frac{l_{bi}}{\rho}\right)
\end{aligned} \tag{241}$$

which can be simplified using conditions (236) to (240):

$$\begin{aligned}
&E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} = \\
&= \underbrace{\frac{1-\tau}{\rho-\mu}x_0\left(1-\frac{l^*_{bi}}{l^*_p}\right)}_{\leq 0} - \underbrace{\frac{1-\tau}{\rho}\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi}-1\right)\left(\frac{x}{x^2_{bi}}\right)^{\beta}(\beta-1)(1-\overline{\kappa}_1^{\frac{1}{\beta}})\left(\frac{\overline{\kappa}_1}{\kappa_1}-1\right)}_{=0} \leq 0
\end{aligned} \tag{242}$$

By condition (231), we have:

$$\Delta CS_{p,bi}(x|\xi < \overline{\xi}_p) \leq 0 \tag{243}$$

▷ **Case (B):** $\xi \in [\overline{\xi}_p, 1]$ **.** I write the parameters κ_2 and $\overline{\kappa}_2$ as

$$\kappa_2 := \frac{\tau - 1}{\tau} \frac{(\beta - 1)\epsilon_p}{1 - \xi} \tag{244}$$

and

$$\overline{\kappa}_2 := \max\{1, \kappa_2\} \tag{245}$$

Using a similar approach as in case (A) but now for strong banks, I get:

$$E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l_{bi}^*}{l_p^*} = \underbrace{\frac{1-\tau}{\rho - \mu} x_0 \left(1 - \frac{l_{bi}^*}{l_p^*}\right)}_{\leq 0} \leq 0 \tag{246}$$

Recalling condition (231), this implies:

$$\Delta CS_{p,bi}(x|\xi \geq \overline{\xi}_p) \leq 0 \tag{247}$$

□

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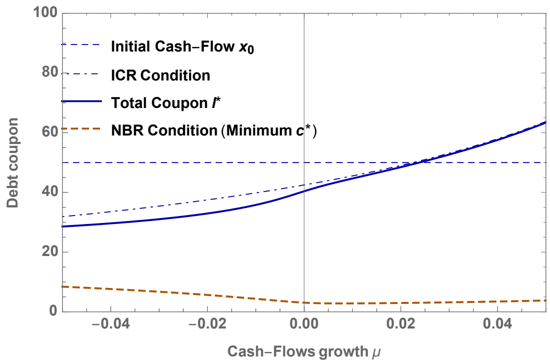
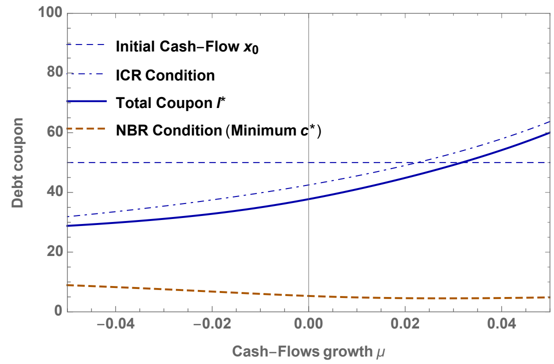
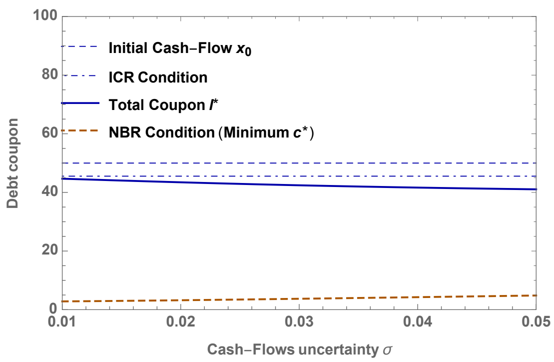


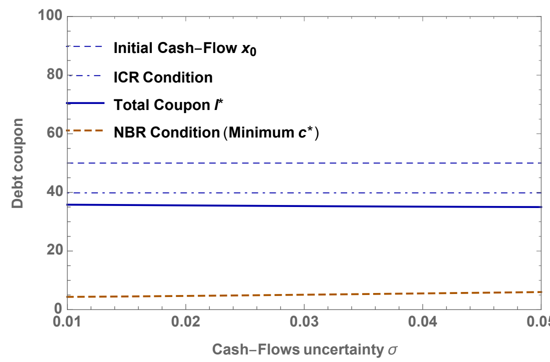
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



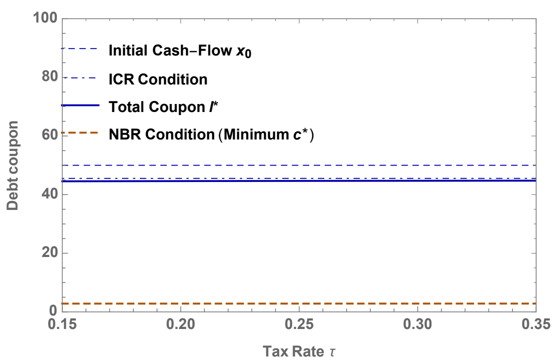
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



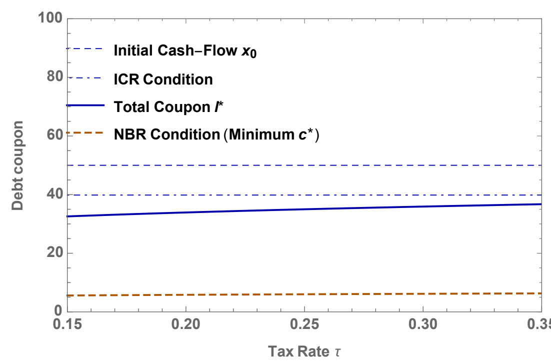
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



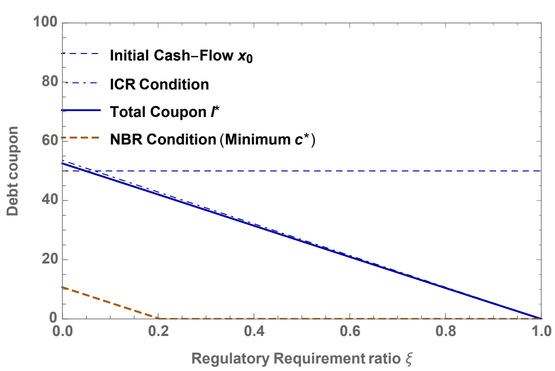
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



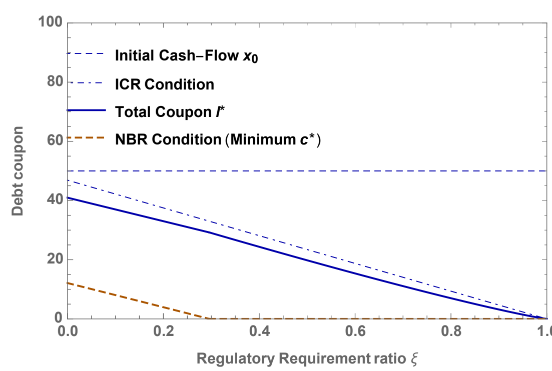
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

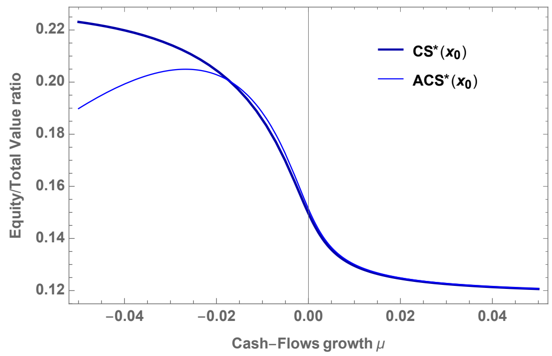


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

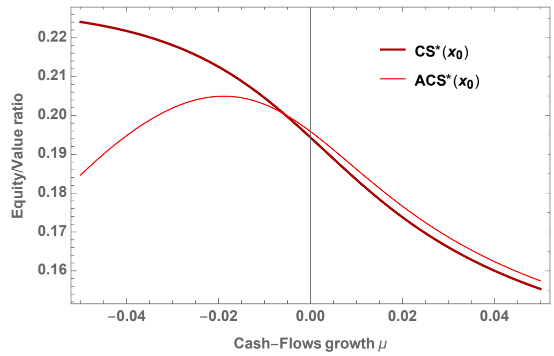


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

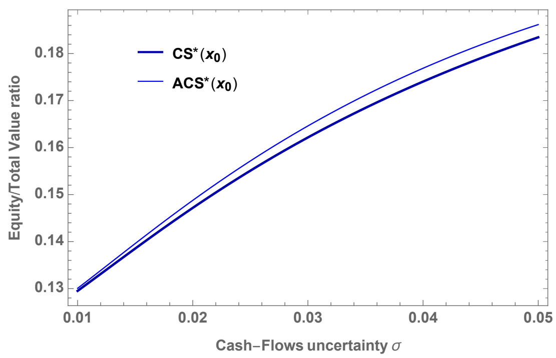
Figure 10: ***Base Case - Equilibrium Coupon Structure. Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon = .2$; $\tau = 0.25$; $x_0 = 50$.



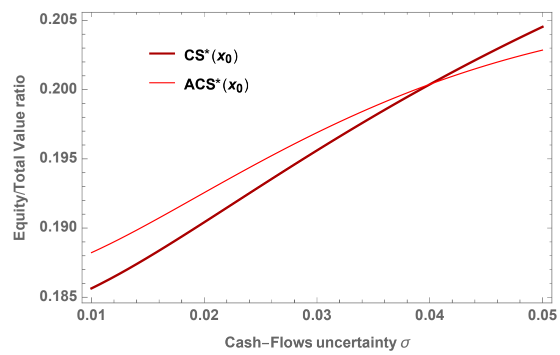
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



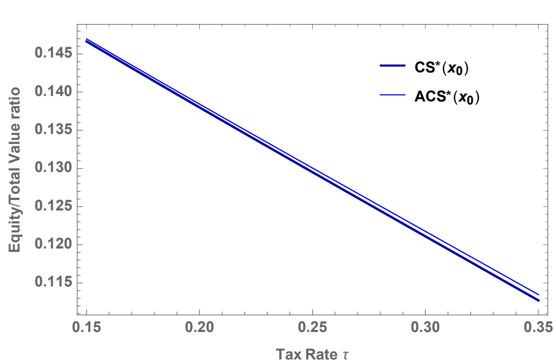
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



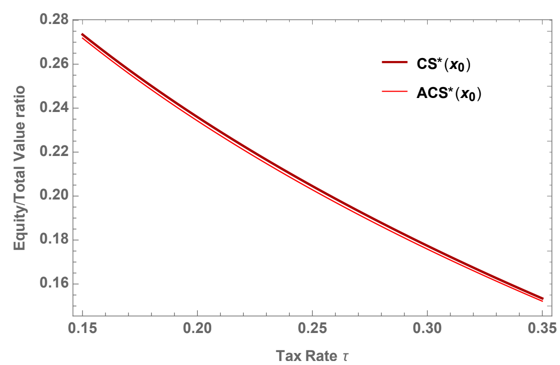
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



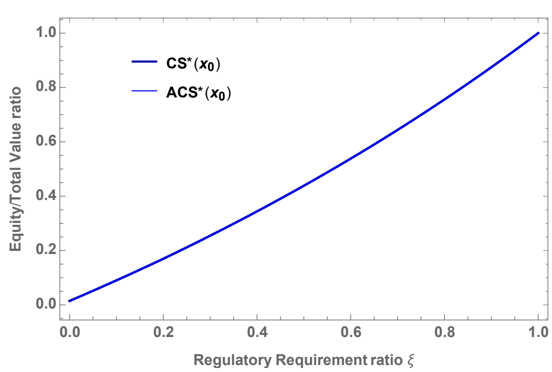
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



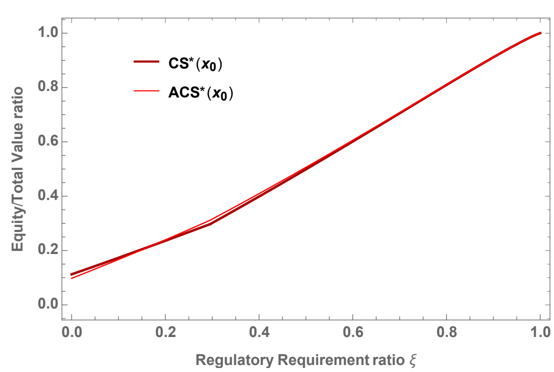
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

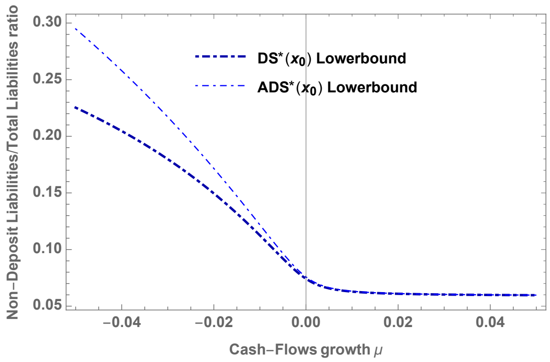


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

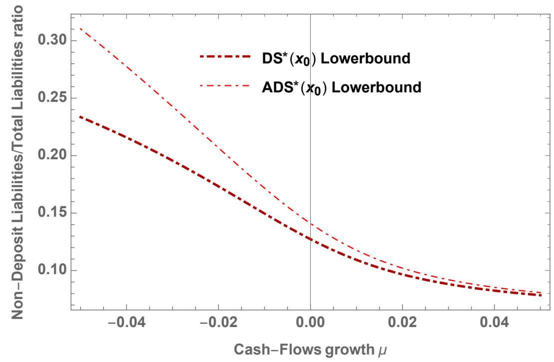


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

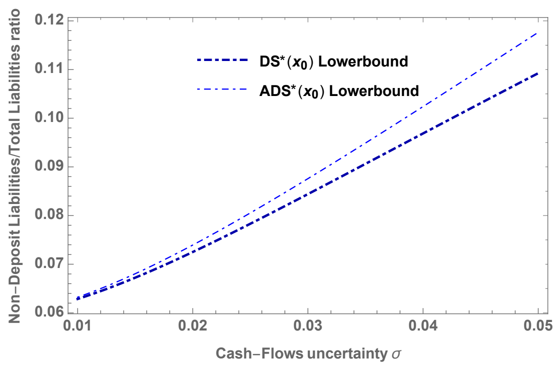
Figure 11: ***Base Case - Equilibrium Capital Structure. Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon = 0.2$; $\tau = 0.25$; $x_0 = 50$.



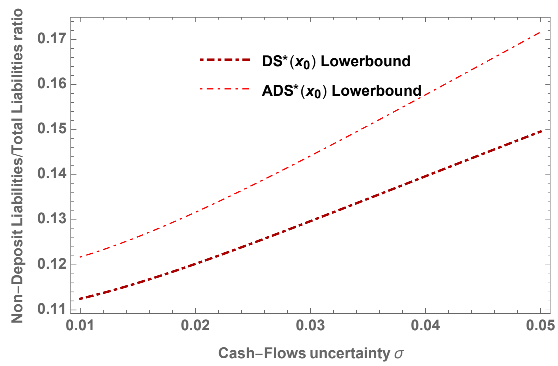
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



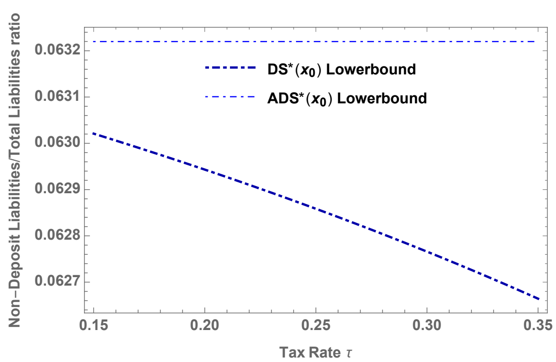
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



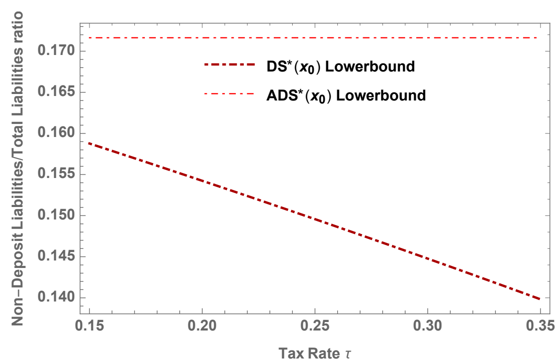
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



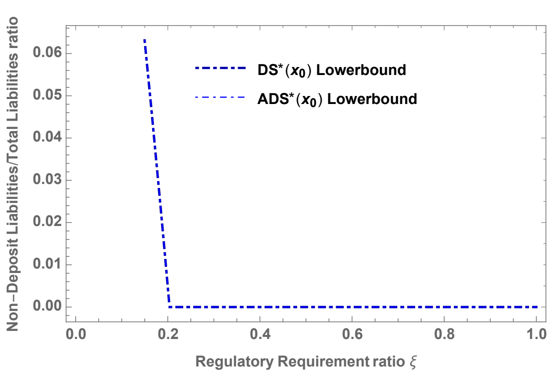
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



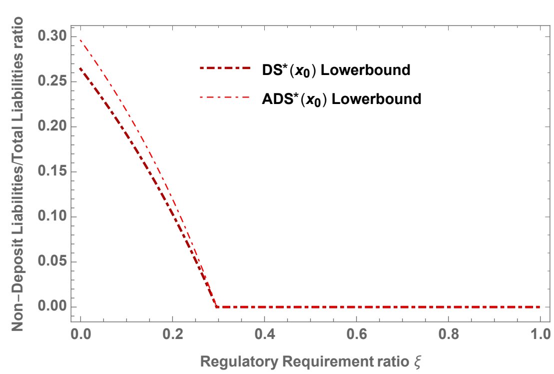
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

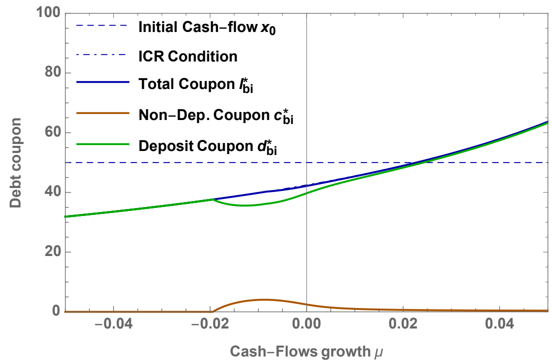


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

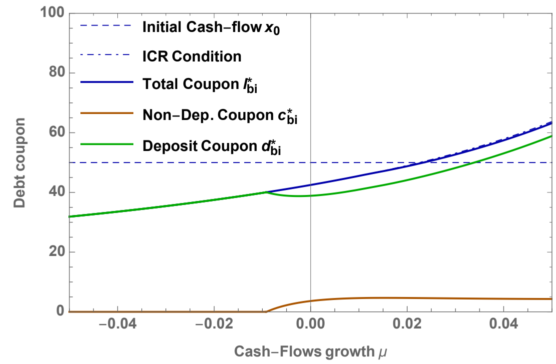


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

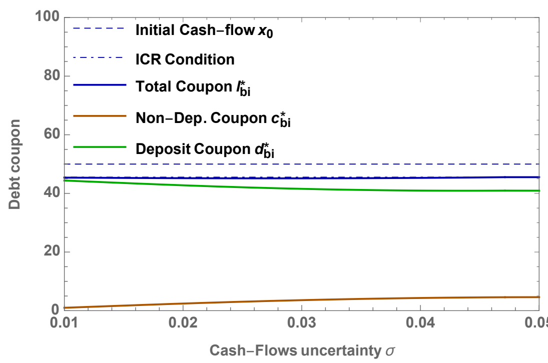
Figure 12: ***Base Case - Equilibrium Debt Structure:*** ***Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon = 0.20$; $\tau = 0.25$; $x_0 = 50$.



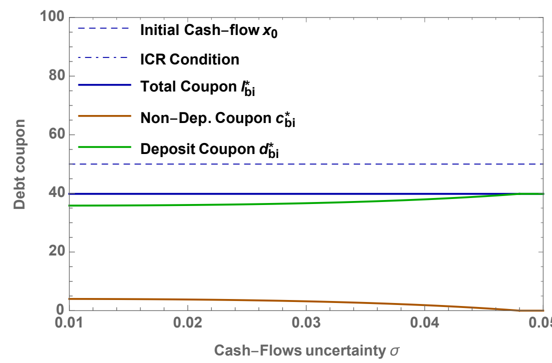
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



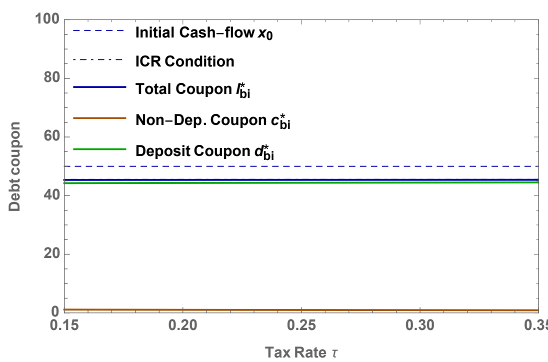
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



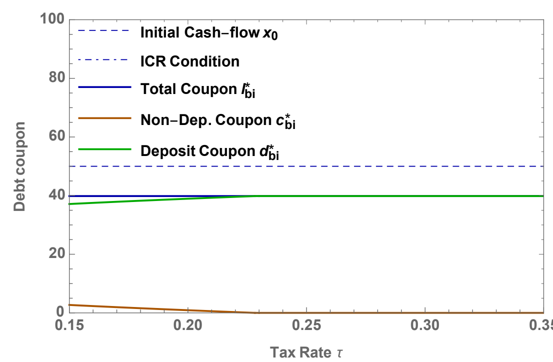
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



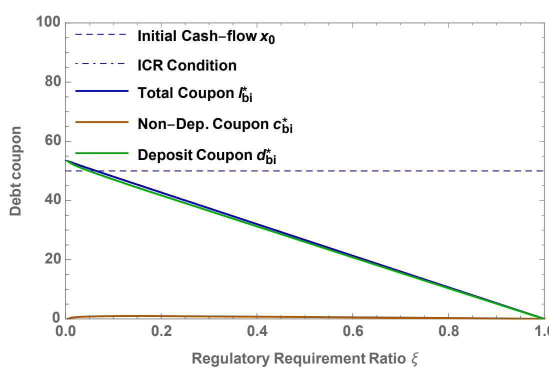
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



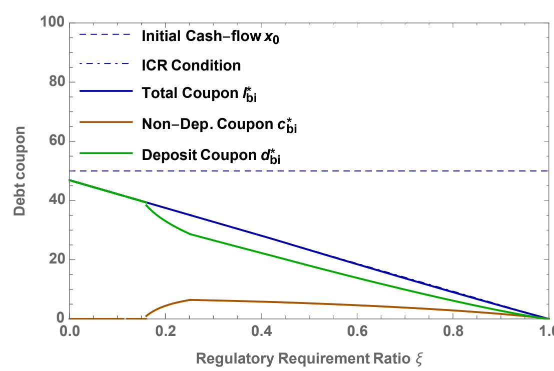
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

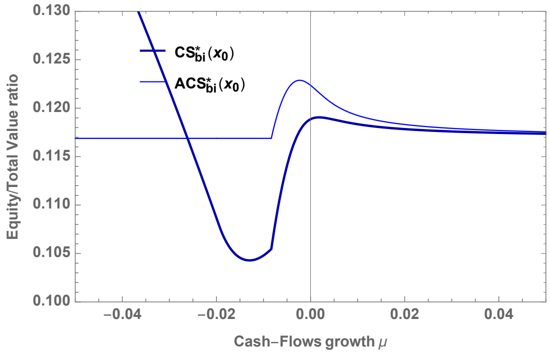


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

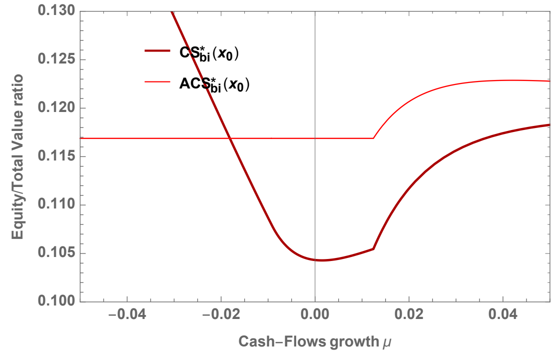


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

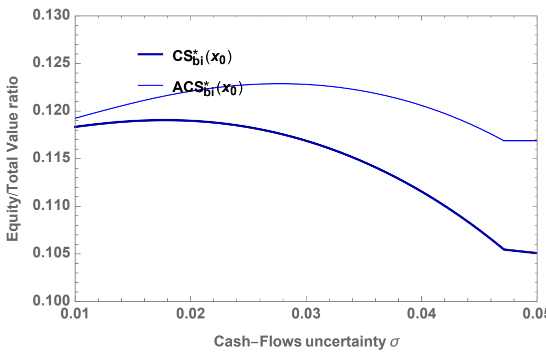
Figure 13: ***Bail-In Case - Equilibrium Coupon Structure. Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



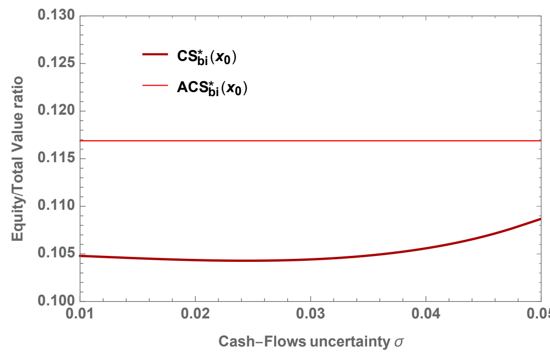
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



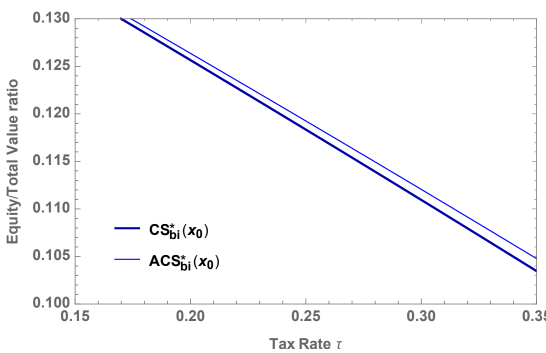
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



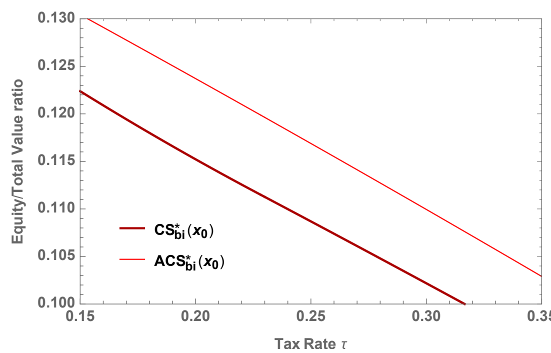
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



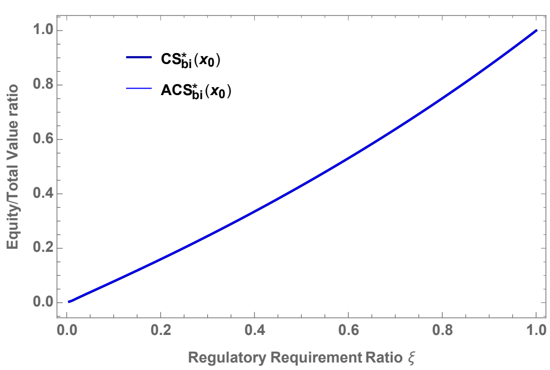
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



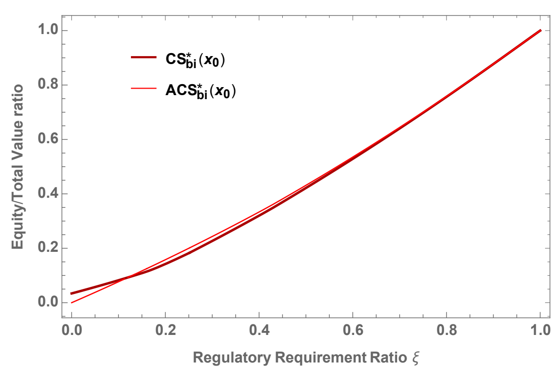
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

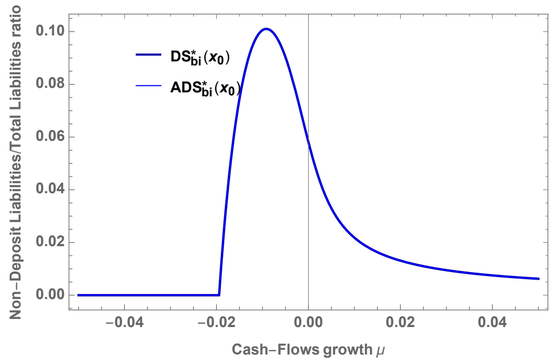


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

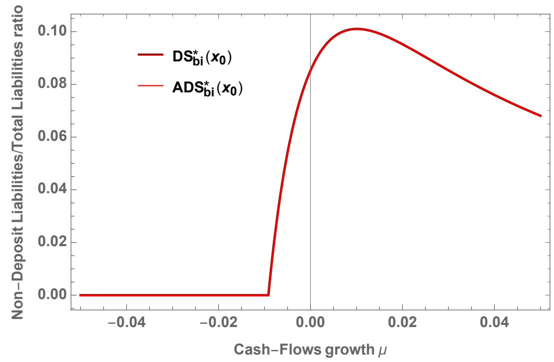


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

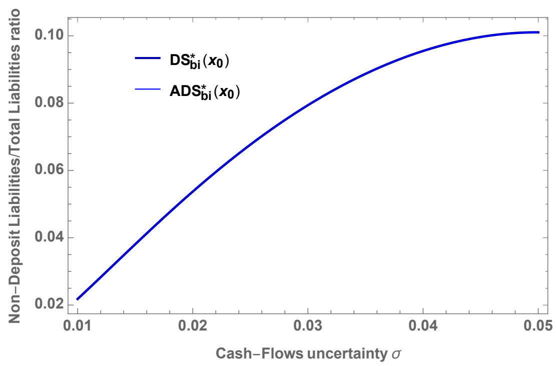
Figure 14: ***Bail-In Case - Equilibrium Capital Structure. Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



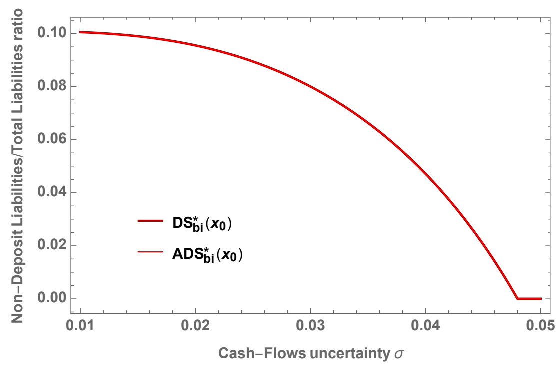
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



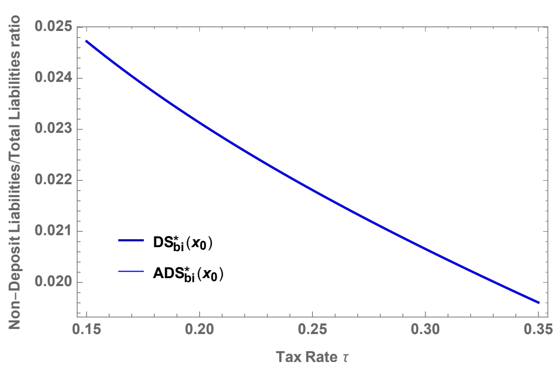
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



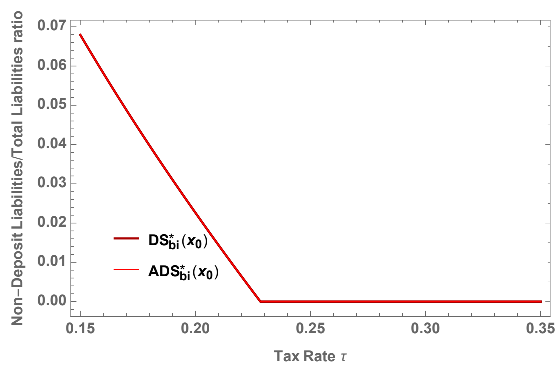
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



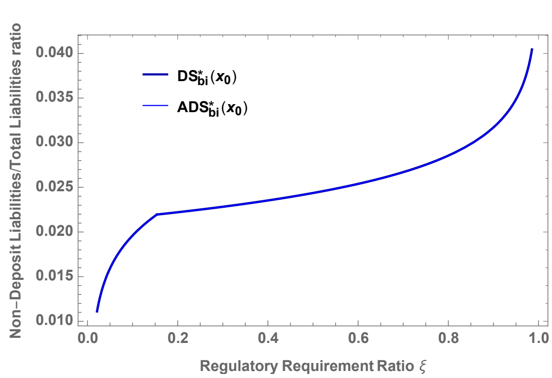
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



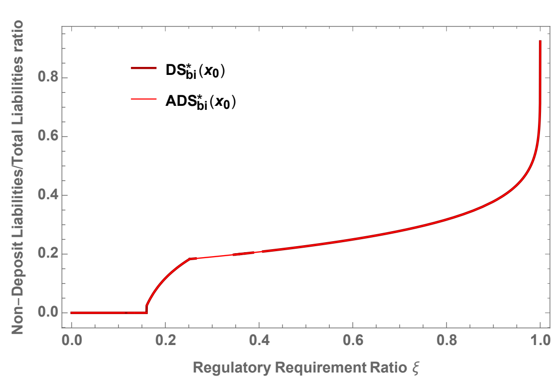
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

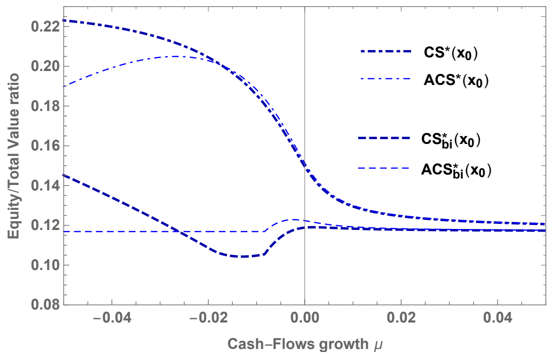


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

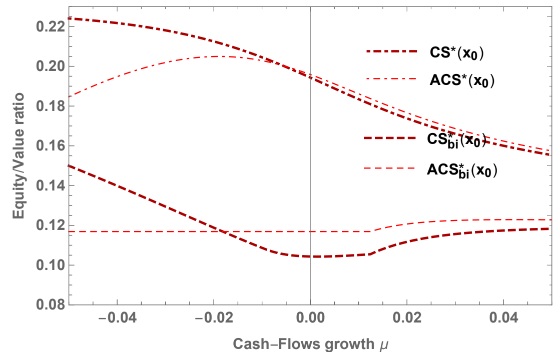


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

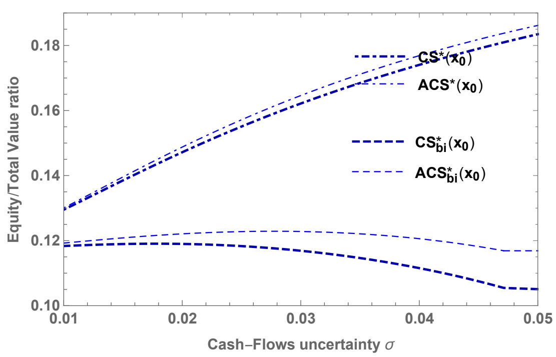
Figure 15: ***Bail-In Case - Equilibrium Debt Structure:*** ***Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



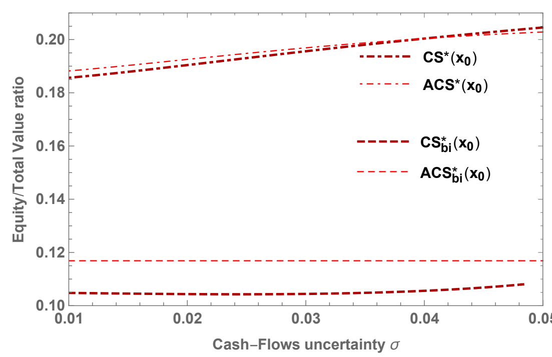
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



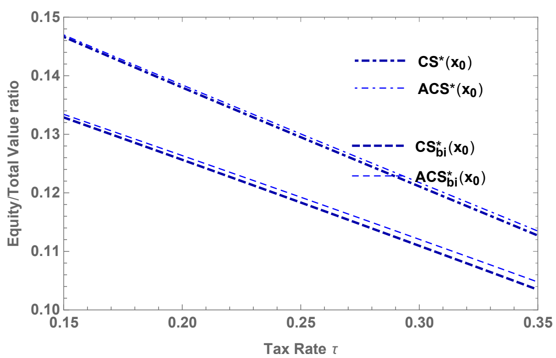
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



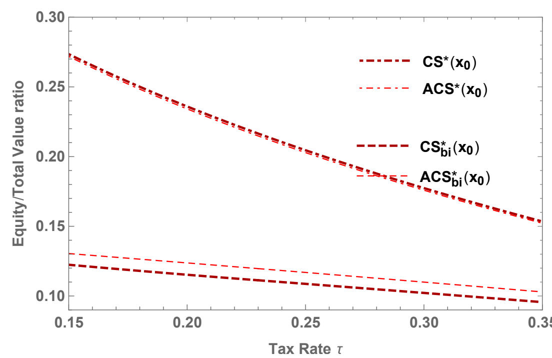
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



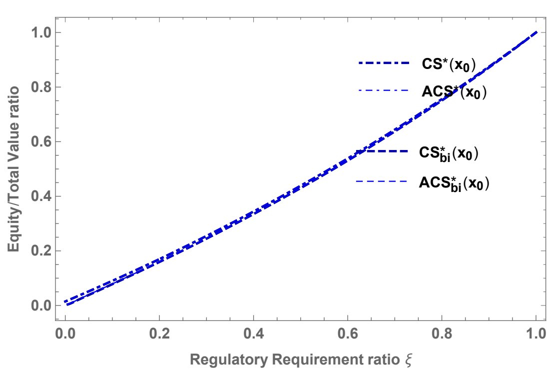
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



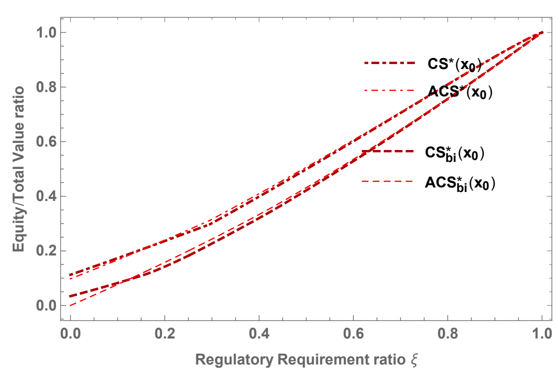
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$

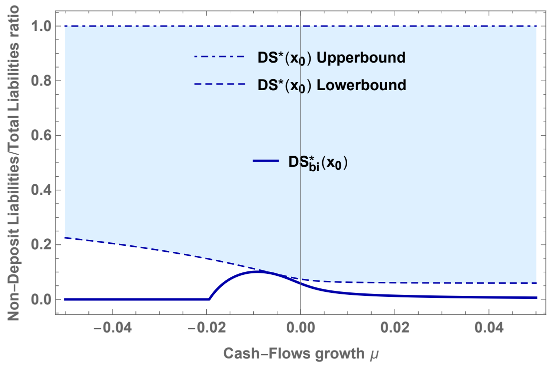


(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$

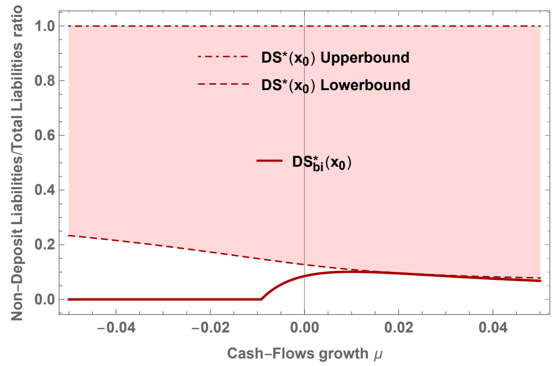


(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

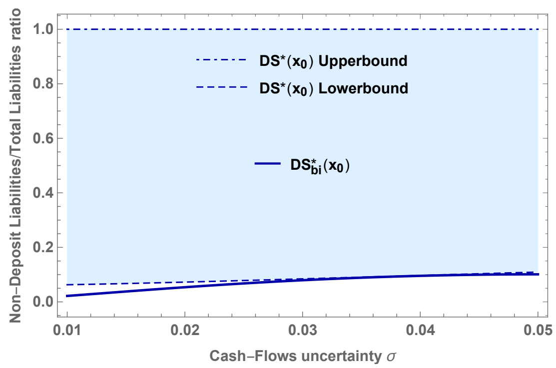
Figure 16: ***Capital Structure Transition:*** *Base Case equilibrium capital structure (dot-dashed lines) vs Bail-In Case equilibrium capital structure (dashed lines).* ***Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$; $\epsilon_p = 0.15$; $\epsilon = 0.2$; $\tau = 0.25$; $x_0 = 50$.



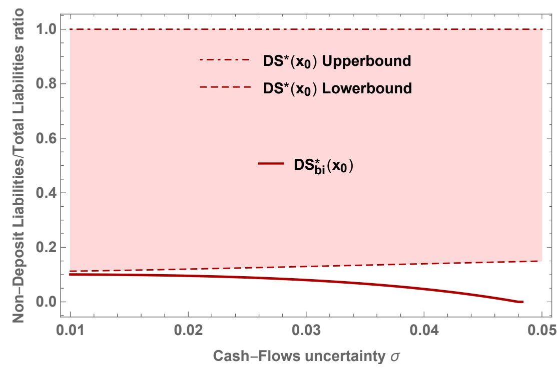
(a) **Good State - Growth change** $\mu \in [-.05, .05]$



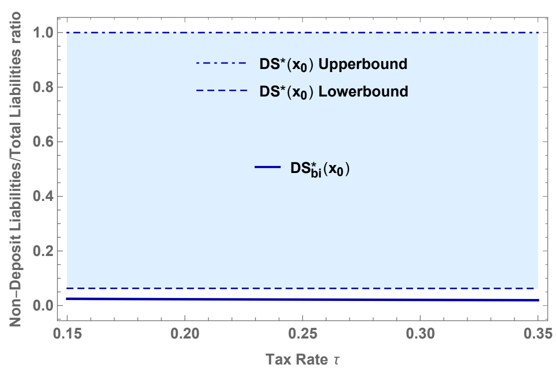
(b) **Bad State - Growth change** $\mu \in [-.05, .05]$



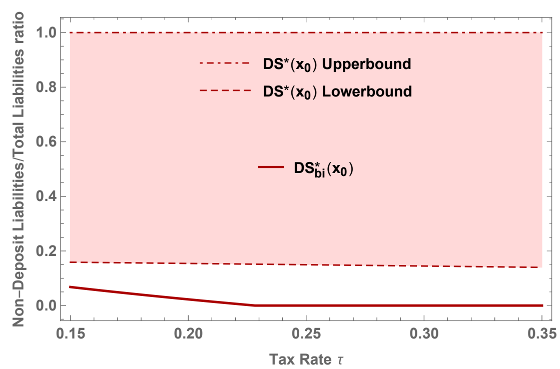
(c) **Good State - Uncertainty change** $\sigma \in [.01, .05]$



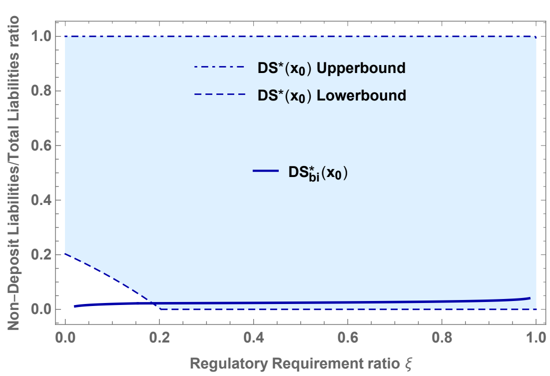
(d) **Bad State - Uncertainty change** $\sigma \in [.01, .05]$



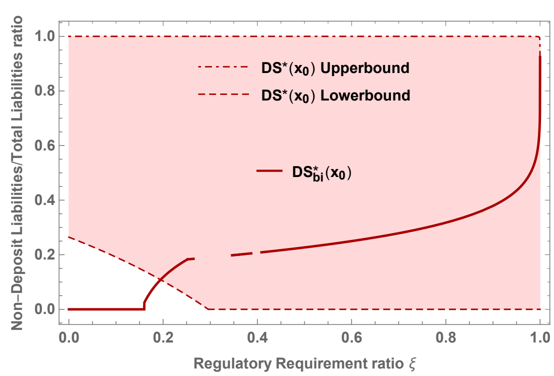
(e) **Good State - Tax changes** $\tau \in [.15, .35]$



(f) **Bad State - Tax changes** $\tau \in [.15, .35]$



(g) **Good State - Cap. Requirements ratio changes** $\xi \in [0, 1]$



(h) **Bad State -Cap. Requirements ratio changes** $\xi \in [0, 1]$

Figure 17: ***Debt Structure Transition:*** *The coloured region is the admissible equilibrium debt structure for the Base Case. The bold line is the Bail-in Case equilibrium Debt Structure.* ***Parameters:*** *Good State:* $\sigma = 0.01$; $\mu = 0.01$; *Bad State:* $\sigma = 0.5$; $\mu = -0.01$; *Common:* $\rho = 0.15$; $\xi = 0.15$;