ASSESSING THE ECONOMY-WIDE EFFECTS OF QUANTITATIVE EASING[*]

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This article examines the macroeconomic impact of the first round of quantitative easing (QE) by the Bank of England. We attempt to quantify the effects of these purchases by focusing on the impact of lower long-term interest rates on the wider economy. We use three different models to estimate the impact of QE on output and inflation: a large Bayesian vector autoregression (VAR), a change-point structural VAR and a time-varying parameter VAR. Our estimates suggest that QE may have had a peak effect on the level of real GDP of around $1\frac{1}{2}\%$ and a peak effect on annual CPI inflation of about $1\frac{1}{4}\%$ points.

The sharp deterioration of the global financial crisis in late 2008 led to the increased risk of a severe downturn on a scale not seen since the Great Depression of the 1930s. In many countries, the fiscal and monetary authorities responded with a variety of conventional and less conventional measures aimed at mitigating the effects on financial stability and the real economy. In the UK, the Bank of England introduced a number of innovative policy measures. As Bean (2011) describes, these measures included enhanced liquidity support, actions to deal with dysfunctional financial markets and large-scale asset purchases. In this article, we focus on assessing the macroeconomic effects of the Bank's programme of large-scale asset purchases (LSAPs), commonly referred to as quantitative easing (QE).

The Monetary Policy Committee (MPC) of the Bank of England announced that it would begin a large programme of asset purchases, mainly of UK government bonds or gilts, on 5 March 2009, at the same time as it reduced Bank Rate, the official UK policy rate, to 0.5%. Despite lowering policy rates close to the zero lower bound (ZLB), the MPC felt that additional measures were necessary to achieve the 2% CPI inflation target in the medium term. The aim of the programme of asset purchases financed by the issuance of central bank money was to inject a large monetary stimulus into the economy, to boost nominal expenditure and thereby increase domestic inflation sufficiently to meet the inflation target. Between March 2009 and the end of January 2010 the Bank purchased a total of £200 billion assets, representing about 14% of UK GDP.

Most of the previous work on this topic has focused on the effects of QE on financial markets (Joyce *et al.*, 2011). Our work by contrast focuses on measuring the wider economic effects of the Bank's asset purchases on output and inflation.

Understanding the effects of QE on the wider economy is of course necessary to appreciate the effectiveness of QE as a policy option during times of financial crisis. It is also useful for understanding the transmission mechanism of unconventional monetary policy.

The approach we take involves conducting counterfactual analysis, to assess what would have happened had QE not been undertaken, which we then compare with a baseline prediction which includes QE. Our analysis is based on three models. We use a large BVAR, estimated over rolling windows, to allow for structural change; an MS-SVAR, where parameters are allowed to change at a particular time to capture regime changes; and a TVP-SVAR, which allows us to assess general time variation in parameters. The BVAR places more weight on past patterns in the data, by incorporating a large data set and the minimum amount of economic structure. Such models have been found to be useful because they allow the analysis of complex interrelationships between a large set of economic data, which in our case involves the interconnections between interest rate spreads and the real economy. The MS-SVAR and the TVP-SVAR employ a small data set but they allow us to incorporate a more sophisticated treatment of structural change. The underlying economic or structural shocks in these models are identified through restrictions on the impulse responses (Baumeister and Benati, 2010).

We use each of the models to conduct counterfactual analysis of the effects of QE. This exercise relies on the empirical evidence in Joyce *et al.* (2011), which suggests that QE reduced medium to long-term government bond yields by about 100 basis points. To produce counterfactual forecasts, we therefore assume that without QE gilt yields would have been 100 basis points higher, *ceteris paribus*. For the purpose of the counterfactual, we also assume that the effects of QE come solely through lower long-term government bond yield spreads. We implement this effect on yields by adjusting the spread between the relevant long-maturity gilt yield in each model and the three-month Treasury bill rate (henceforth the government bond spread). The link between government bond spreads and macroeconomic variables is given a structural interpretation in, for example, Estrella (2005). One caveat here is that our models do not allow us to discriminate between the effects of movements in bond spreads that come through term premia and those that come through expected future policy rates. So to the extent that QE effects on spreads come mainly through term premia (as much of the literature suggests – see Section 2), and this has different macroeconomic effects to spread movements caused by future policy rate expectations, this will not be captured in our analysis.

Our multiple models strategy is similar in spirit to the approach adopted by Chung *et al.* (2012) in their analysis of the incidence of the ZLB interest rate policy environment (though their article only uses one model in its analysis of the Federal Reserve's LSAP programmes). Bridges and Thomas (2012) also use a number of monetary models to estimate the effect of the Bank of England asset purchases on the level of GDP and CPI inflation. The use of different models that vary in their emphasis on data *versus* economic structure increases our faith in the likely robustness of our conclusions. Our analysis encompasses existing time-series models in the literature on the effects of unconventional monetary policy (Baumeister and Benati, 2010; Lenza *et al.*,

2010; Joyce *et al.*, 2011) and extends this literature by including models that account for structural change.

Results from the counterfactual analysis of the effects of QE using the large BVAR model suggests that without QE there would have been larger declines in real GDP during 2009 and CPI inflation would have been low or even negative. The QE policy was therefore effective in helping the UK economy avoid a deeper recession and deflation. The MS-SVAR and TVP-SVAR models provide similar evidence, if anything suggesting that QE had even larger effects on output and inflation.

The rest of this article is structured as follows. Section 1 discusses the UK's experience with QE and Section 2 reviews some of the related literature on the effects of QE and other large-scale asset purchases on financial and macro variables. Our econometric framework is described in Section 3, the data are described in Section 4, and the counterfactual assumptions we use in our analysis are discussed in Section 5. Section 6 presents empirical results for each of the models. Section 7 provides a summary of the key results and Section 8 notes the caveats to our analysis and Section 9 concludes.

1. Quantitative Easing in the UK

Large-scale asset purchases in the UK were a culmination of a package of measures designed to address the consequences of the financial crisis.[1] These measures included the provision of enhanced liquidity support, measures to enhance market functioning and QE or large-scale asset purchases (Bean, 2011). The provision of liquidity support was centred on the £185 billion Special Liquidity Scheme introduced in April 2008, which allowed banks to swap mortgage-backed securities and other illiquid assets for Treasury bills. A Discount Window Facility was also introduced to meet the short-term liquidity needs of financial institutions requiring assistance. In addition, there was the assurance that the Bank of England was ready to offer emergency liquidity support at a penalty rate and against a broader range of collateral to otherwise solvent financial institutions that were experiencing liquidity problems. To address market functioning, an Asset Purchase Facility was created to allow the Bank of England to purchase high-quality commercial article and sterling investment-grade corporate bonds. Before the QE policy was introduced, these purchases were financed by the issuance of Treasury bills and the cash management operations of the Debt Management Office. Like the offer of emergency liquidity support, the knowledge that the central bank was wishing to purchase these assets may have improved overall market functioning.

The QE policy was first announced in March 2009 and it involved the Bank of England buying assets, mainly UK government bonds (gilts), financed by the issuance of central bank reserves. The effect of these purchases was to reduce gilt yields and to stimulate demand through a number of channels. In normal times, reducing Bank Rate would be the policy chosen to address demand shocks. However, with Bank Rate close to the ZLB and given the risk of undershooting the inflation target, the Bank of

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[1] Aït-Sahalia *et al.* (2009) and Lenza *et al.* (2010), among others, provide a review of the various measures used by major central banks in response to the Great Financial Crisis of 2008.

England's MPC had to use unconventional methods to ease monetary conditions further. The initial MPC decision was for the Bank to make £75 billion of asset purchases. This was extended subsequently to a total of £125 billion in May 2009, to a total of £175 billion in August 2009 and to a total of £200 billion in November 2009, with the purchases completed at the end of January 2010.[2] This represented about 14% of annual UK GDP.

Although there are a number of possible channels through which QE may affect the wider economy (see discussion in Benford *et al.*, 2009), most analysis has emphasised the so-called portfolio balance channel. This mechanism operates through QE purchases bidding up the prices of gilts and other assets that are more substitutable for gilts than money, and this in turn stimulates demand through lower borrowing costs and wealth effects. Portfolio balance models as described by Tobin (1969), among others, were used by Joyce *et al.* (2011) to determine the financial market impact of QE. They find that the predictions of these models are broadly consistent with the event study evidence for the UK. We discuss this and other empirical evidence in the next Section.

2. Related Literature

Most of the literature on the effects of QE and the use of other unconventional monetary instruments has focused on financial market variables, as opposed to the effects on real activity or inflation. This is primarily due to the difficulty of identifying the appropriate counterfactual. In this Section, we summarise the literature on the effects of QE or LSAPs on financial and real variables, both in the UK and in other countries.

The assessment of the effects of non-standard monetary policies on financial variables has mainly relied on event study methods. Bernanke *et al.* (2004), for example, provide a comprehensive analysis of financial market reactions to various non-standard Fed policy announcements that altered the relative supply of US Treasury securities. They conclude that both changes in relative asset quantities and the expectation of such changes have had an impact on yields or asset returns. These results are supported by vector autoregressions (VAR)-based term structure models. Bernanke *et al.* (2004) also provide some evidence that QE as implemented by the Bank of Japan (providing excess reserves to maintain the interest rate at zero and open market purchases of government bonds) may have generated lower yields over the QE period, although there is weaker support from event studies compared with those for the US.[3]

Gagnon *et al.* (2011) provide an assessment of the first round of LSAPs conducted by the Fed in the wake of the great financial crisis (commonly referred to as LSAP1). On the basis of event studies of LSAP announcements, they suggest there was a contraction in Treasury yields and yields on mortgage-backed securities (MBS) of about 90 and 110 basis points respectively. They suggest that the decline in long-term interest rates

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[2] The analysis in the article is for the first round of asset purchases by the Bank of England. The Bank began a second round asset purchases in October 2011.
[3] A comprehensive review of the financial and macroeconomic effects of QE in Japan is provided by Ugai (2007).

largely reflected the fall in risk premia generated by these purchases, mainly through the reduction of duration risk. They also use a time-series econometric model of asset quantities estimated on the basis of pre-crisis data to determine the impact of LSAPs, which suggests slightly smaller effects. Using a different approach based on panel data analysis of individual bonds, D'Amico and King (2010) find that LSAP1 had an effect on longer term Treasury yields of about 30 basis points for the 5 to 15-year sector. Krishnamurthy and Vissing-Jorgensen (2011) examine both LSAP1 and the second round of Fed purchases (LSAP2) using an event study approach. They find evidence of a large decline in interest rates in the first episode but not the second (though this may reflect the fact that the markets had already priced in much of the expected impact before the second programme was announced). They identify a number of different channels through which QE may work, such as duration, liquidity and the long-term safety channel. Swanson (2011) revisits the Operation Twist experiment of the 1960s using event study techniques and argues that it was broadly comparable in scale to LSAP2. He finds that both policies reduced longer term Treasury yields by around 15 basis points.[4]

The UK's experience with QE in the recent financial crisis has been documented in a number of studies. Meier (2009), Bean *et al.* (2010), Dale (2010) and Joyce *et al.* (2011), among others, have discussed the operational details of large-scale asset purchases by the Bank of England and analysed various aspects of the impact of these unconventional monetary measures. Meier (2009) used an event studies approach to assess the impact of QE announcements and suggests long-term government bond yields declined between 40 and 100 basis points following the initial QE announcement by the Bank of England in March 2009. Joyce *et al.* (2011) provide a more comprehensive assessment using event studies and portfolio balance models. In this framework, it is assumed that gilts and money are imperfectly substitutable assets and a multiplier calculated from a Markowitz–Tobin portfolio choice-type model (Markowitz, 1952) determines the effects of changes in the quantity of gilts on excess asset returns in a portfolio with money, equities, corporate bonds and gilts.[5] They suggest that QE lowered long-term gilt yields by about 100 basis points and that most of the decline was generated by portfolio balance effects.

There are far fewer studies that try to estimate the macroeconomic effects of unconventional monetary policy measures. One of the first in the current crisis was by Lenza *et al.* (2010), who conduct a comprehensive review of the European Central Bank's use of non-standard monetary instruments in response to the crisis. The ECB embarked on an 'enhanced credit support' programme (Trichet, 2009), focused on market liquidity, from mid-2009 to mid-2010 in addition to a multitude of other measures intended to enhance market functioning introduced at the onset of the crisis. Lenza *et al.* (2010) provide evidence, based on a counterfactual analysis using a large BVAR model, that these measures were successful in reducing financial market

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[4] Other supportive evidence on the effects of the Fed's LSAPs programme is provided by Hamilton and Wu (2012) and Doh (2010).

[5] Others have also used portfolio balance models to estimate the impact of LSAPs. (for example, Kimura and Small, 2006) who suggested that QE in Japan had some positive portfolio balance effects and reduced risk premia on some assets. Neely (2010) used a portfolio balance model to examine the international effects of US LSAPs.

dysfunction given the noticeable contraction in money market spreads. They also find that these measures had a positive effect on output and inflation but with a lag. Another VAR-based study by Baumeister and Benati (2010) provides evidence of a significant macroeconomic impact in the US, UK and the euro area due to the observed decrease in long-term bond spreads following asset purchases.

The impact of the Fed's LSAPs on the US macroeconomy is also covered in Chung *et al.* (2012), who find that the LSAPs were successful. In particular, simulations from the Fed's FRB/US macroeconomic model suggest that asset purchases prevented deflation in the US and reduced the rate of unemployment. The authors suggest that the boost to the level of real GDP was about 3%, inflation was 1% higher and that the unemployment rate was reduced by 1.5% points compared with what it would otherwise have been.

The theoretical underpinnings for expecting changes in asset quantities to affect yields are provided in Vayanos and Vila (2009), who develop a model based on investors with preferred habitats (Greenwood and Vayanos, 2008, offer empirical evidence in support of the model's predictions).[6] However, in most conventional New Keynesian models, QE has no wider economic effects, unless it changes agents' expectations about the future path of interest rates through the signalling channel. Eggertsson and Woodford (2003) argue that there are no portfolio balance effects in these models because the reduction in private sector portfolio risks resulting from central bank asset purchases is offset by a corresponding increase in the riskiness of public sector portfolio due to the inherent uncertainty of future taxes and spending, making QE purchases irrelevant through this channel. However, the literature incorporating the use of unconventional monetary policies into theoretical macroeconomic models is steadily evolving. In more recent work, Curdia and Woodford (2009) suggest that there can be some role for credit easing, which involves changing the composition of assets on a central bank's balance sheet but not for QE, which would still be ineffective at the ZLB.[7] But when financial frictions or incomplete markets are coupled with imperfect asset substitutability, changing the maturity structure of assets can also affect asset prices. A useful starting point is the inclusion of Tobin's idea of imperfect asset substitutability in standard New Keynesian models. For example, Andrés *et al.* (2004) and Harrison (2012) develop microfoundations for preferred-habitats and portfolio balance effects which is supportive of a role for QE within a dynamic stochastic general equilibrium (DSGE) framework. In general, to explain the macroeconomic effects of QE and other unconventional monetary policies fully, the (modified) DSGE model must capture the frictions that generate interest rate spreads and linkages between interest rate spreads and the real economy. An insightful overview of related issues in this emerging literature is found in, for example, Christiano (2011).

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[6] Analysis of the effects of altering the maturity structure of government debt is not new. Informal analysis of the preferred-habitat theory and empirical evidence on debt maturity structure are available in, for example, Modigliani and Sutch (1966, 1967).

[7] Christiano and Ikeda (2011) also consider the role of credit easing using theoretical models with financial frictions. 'Credit easing' in the US is described as the Fed's purchases of mortgage-backed securities, which changed the composition of assets on the Fed's balance sheet.

3. Econometric Framework

In this Section, we describe the econometric models used in this article.

3.1. *Bayesian VAR (BVAR)*

The seminal work by Sims (1980) introduced the use of VAR into macroeconometric modelling and VAR models continue to occupy centre stage. In this article, we use Bayesian methods to estimate them. Specifically, we estimate a large BVAR model similar to the model employed by Lenza *et al.* (2010) As our analysis involves a large data set, BVAR models are useful to overcome parametrisation problems which would otherwise be encountered when a standard VAR is estimated in large dimensions. The BVAR model allows us to use *a priori* information to restrict the parameter space. Our approach of applying prior information to a standard VAR model can be motivated from both a Bayesian and a classical perspective. We view the use of Bayesian technology as desirable mainly on pragmatic rather than philosophical grounds.

3.1.1. *Notation and preliminaries*

The model belongs to the general class of BVAR models for large data sets.[8] Assuming that all the variables in the large data set are in the vector $\mathbf{Y}_t$, we can write the model as follows:

$$\mathbf{Y}_t = \mathbf{\Theta}_0 + \mathbf{\Theta}_1\mathbf{Y}_{t-1} + \cdots + \mathbf{\Theta}_p\mathbf{Y}_{t-p} + \mathbf{e}_t, \tag{1}$$

where $\mathbf{e}_t$ is a vector white-noise error term, $\mathbf{\Theta}_0$ is a vector of constants and $\mathbf{\Theta}_1$ to $\mathbf{\Theta}_p$ are parameter matrices.

3.1.2. *A normal-inverted Wishart AR(1) prior*

As will be discussed later, our large data set comprises macroeconomic and financial market variables. A good prior for BVAR models of the macroeconomy is a simple random walk forecast; see, for example, Litterman (1986). Many macroeconomic and financial market variables are characterised by persistent processes. In general, simple autoregressive (AR) or random walk (RW) models are known to produce reasonable forecasts for macroeconomic and financial variables (over short horizons). We therefore choose a univariate AR(1) process with high persistence as our prior for each of the variables in the BVAR model. With this prior, the 'own' first lag is considered to be the most important in every equation in the BVAR. Specifically, the expected value of the matrix $\mathbf{\Theta}_1$ is $E(\mathbf{\Theta}_1) = 0.99 \times \mathbf{I}$. We assume that $\mathbf{\Theta}_1$ is conditionally (on $\mathbf{\Sigma}$) normal, with first and second moments given by

$$E[\Theta_1^{(ij)}] = \begin{cases} 0.99 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, \quad \text{Var}[\Theta_1^{(ij)}] = \phi\sigma_i^2/\sigma_j^2, \tag{2}$$

where $\Theta_1^{(ij)}$ denotes the element in position (i,j) in the matrix $\mathbf{\Theta}_1$, and where the covariances among the coefficients in $\mathbf{\Theta}_1$ are zero. The shrinkage parameter ϕ

[8] See, for example, Banbura *et al.* (2010).

determines the tightness of the prior or the extent to which the data influence the estimates. With a tight prior, the data have little or no influence on the estimates as $\phi \to 0$. For a loose prior, where $\phi \to \infty$ there is an increasing role for the data and the estimates then converge to the standard *OLS* estimates. To complete the specification of our BVAR prior, we assume that the constant, $\boldsymbol{\Theta}_0$, has a diffuse normal prior and the matrix of disturbances has an inverted Wishart prior, $\boldsymbol{\Sigma} \sim \mathrm{iW}(v_0, S_0)$. v_0 and S_0 are the prior scale and shape parameters with the expectation of $\boldsymbol{\Sigma}$ equal to a fixed diagonal residual variance $\mathrm{E}(\boldsymbol{\Sigma}) = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_N^2)$. This is a conjugate prior with a normal-inverted Wishart posterior distribution. The BVAR model is estimated using rolling windows to account for structural change. Additional technical information on model estimation and prior tightness is provided in Appendix A.

3.2. *Change-point SVAR (MS-SVAR)*

Our consideration of regime changes is motivated by the fact that since the early 1970s, the UK monetary policy regime has changed a number of times. Since how agents form their expectations will have changed under different monetary policy reaction functions, macroeconomic dynamics over this period cannot be easily described by deep parameters of a single structural model. Since the collapse of the Bretton Woods system in the early 1970s, we might loosely identify four successive monetary policy regimes in the UK. These are monetary targeting (not explicitly introduced until 1979 but monetary aggregates were monitored from the mid-1970s), an informal exchange rate targeting regime in the late-1980s and membership of the exchange rate mechanism (ERM) from 1990–2, inflation targeting after 1992 and more recently inflation targeting in a ZLB environment with the use of unconventional monetary policy instruments.[9] The use of four different structural models might help us to understand agents' actions inside these regimes but it would not be able to capture agents' expectations that the policy regime might change in the future.

The following MS-SVAR model allows us to model (in a reduced-form manner) changes in the policymaker's reaction function and to study how aggregate dynamics have been affected.

$$\mathbf{Y}_t = \mathbf{c}_S + \sum_{j=1}^{K} \mathbf{B}_{j,S} \mathbf{Y}_{t-j} + \mathbf{A}_{0,S} \varepsilon_t, \tag{3}$$

where the data vector $\mathbf{Y}_t$ contains monthly data on the three-month Treasury bill (R_t), the 10-year government bond yield spread (S_t) (defined as the 10-year government bond yield minus the three-month Treasury bill rate), annual GDP growth (y_t), annual CPI inflation (π_t), annual M4 growth (M_t) and the annual change in stock prices (SP_t); $\mathbf{B}_{j,S}$ and $\mathbf{A}_{0,S}$ are regime-dependent autoregressive coefficients and structural shock loading matrices respectively.

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[9] The Bank of England was given operational independence for monetary policy in 1997. However, the UK has had an inflation target since late 1992 and the Bank held joint meetings with the Treasury ahead of policy decisions by the Chancellor of the Exchequer.

As explained in Chib (1998), the dates of (say, *M*) regime breaks in the model are unknown and they are modelled *via* the latent state variable *S*, which is assumed to follow an (*M*-state) Markov chain process with restricted transition probabilities $p_{ij} = p(S_t = j \mid S_{t-1} = i)$ given by

$$
\begin{aligned}
p_{ij} &> 0 \text{ if } i = j \\
p_{ij} &> 0 \text{ if } j = i+1 \\
p_{MM} &= 1 \\
p_{ij} &= 0 \text{ otherwise.}
\end{aligned}
\tag{4}
$$

For example, if $M = 4$ the transition matrix is defined as

$$
\tilde{\mathbf{P}} = \begin{pmatrix}
p_{11} & 0 & 0 & 0 \\
1 - p_{11} & p_{22} & 0 & 0 \\
0 & 1 - p_{22} & p_{33} & 0 \\
0 & 0 & 1 - p_{33} & 1
\end{pmatrix}.
$$

Equations (3) and (4) define a Markov-switching VAR with non-recurrent states where transitions are allowed in a sequential manner. For example, to move from Regime 1 to Regime 3, the process has to visit Regime 2. Similarly, transitions to past regimes are not allowed. This imposed structure (which is not necessarily more restrictive than a standard Markov-switching model) implies that any new regimes are given a new label (rather than being explicitly linked to past states as in a standard Markov-switching model) and this allows us to isolate periods of interest (such as the current period) and tailor our shock identification scheme accordingly.

3.2.1. *Identification of structural shocks*

In this model, we identify four structural shocks: a monetary policy shock, a demand shock, a supply shock and a shock to the yield spread. Following Baumeister and Benati (2010), these shocks are identified using a combination of sign and zero type restrictions; see Table 1.

We impose standard sign restrictions for the monetary policy, demand and supply shocks. A positive monetary policy shock, which increases the short-term rate, will lead to a compression in the yield spread, lower GDP growth rate and lower inflation. A positive demand shock will lead to higher inflation and output, short-term interest rates, money growth and stock prices, whereas a negative supply shock will lead to

Table 1
Sign Restrictions − MS-SVAR Model

Shocks\Variables	R_t	S_t	Y_t	π_t	M_t	SP_t
Monetary policy	≥ 0	$\leq$	$\leq$	$\leq$	?	?
Spread		$\geq$	$\leq$	$\leq$	?	?
Demand	$\geq$	?	$\geq$	$\geq$	$\geq$	$\geq$
Supply	?	?	$\leq$	$\geq$	?	?

Notes. For variable definitions see discussion of (3).

higher inflation and lower output growth. On the other hand, a negative shock to the yield spread is assumed to have zero contemporaneous impact on the short-term interest rate but leads to lower inflation and output growth.

The MS-SVAR not only accounts for different policy regimes but we are also able to examine the effects of the policymaker's inability to change the interest rates to stimulate demand, as under the ZLB. We only do this in the most recent regime by imposing the prior assumption that the policy rate does not depend on other lagged endogenous variables. We show below that our benchmark model estimate of Regime 4 roughly coincides with the 2007–9 financial crisis (Figure 1).

3.3. *Time-varying Parameter SVAR (TVP-SVAR)*

Another model that captures policy regime changes is the following TVP-SVAR:

$$\mathbf{Y_t} = \mathbf{c_t} + \sum_{l=1}^{L} \boldsymbol{\phi}_{l,t}\mathbf{Y}_{t-l} + \mathbf{v_t}, \tag{5}$$

where $\mathbf{Y_t}$ contains quarterly data on the three-month Treasury bill (R_t), the 10-year government bond yield spread (S_t) (defined as the 10-year government bond yield minus the three-month Treasury bill rate), annual GDP growth (y_t) and annual CPI inflation (π_t).
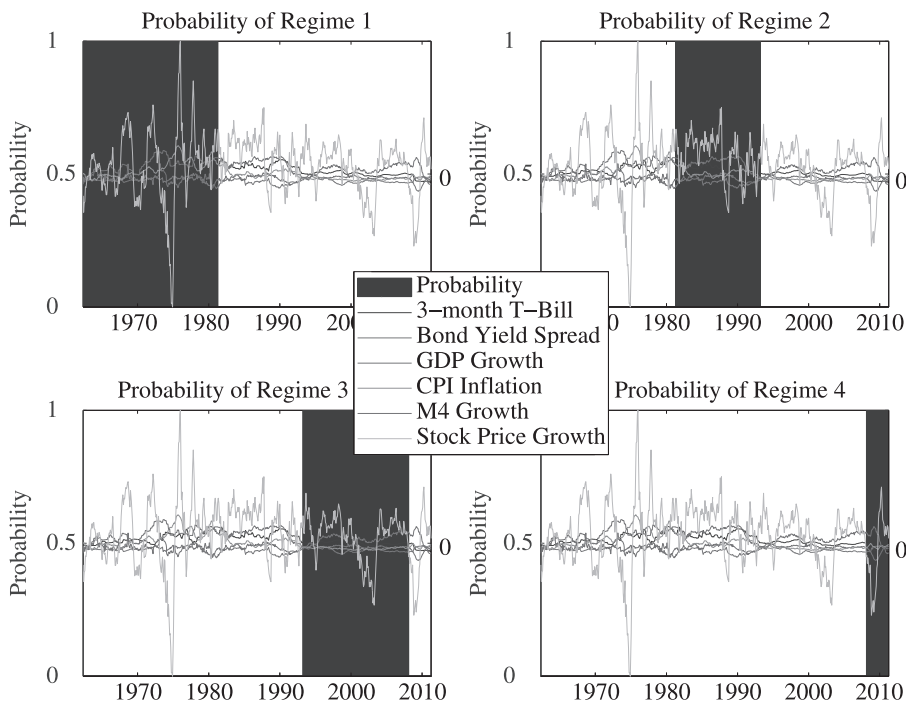

Fig. 1. *Regime Estimation for the MS-SVAR Model*

The law of motion for the coefficients is given by

$$\tilde{\phi}_{l,t} = \tilde{\phi}_{l,t-1} + \eta_t, \tag{6}$$

where $\tilde{\phi}_{l,t} = \{c_t, \phi_{l,t}\}$. As in Cogley and Sargent (2005), the covariance matrix of the innovations $\mathbf{v_t}$ is factored as

$$\mathrm{E}\left(\mathbf{v}_t\mathbf{v}_t'\right) \equiv \mathbf{\Omega}_t = \mathbf{A}_t^{-1}\mathbf{H}_t(\mathbf{A}_t^{-1})'. \tag{7}$$

The time-varying matrices $\mathbf{H}_t$ and $\mathbf{A}_t$ are defined as

$$\mathbf{H}_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 \\ 0 & 0 & 0 & h_{4,t} \end{bmatrix} \quad \mathbf{A}_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 \end{bmatrix}, \tag{8}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + \tilde{v}_t.$$

Following Primiceri (2005), we postulate the non-zero and non-one elements of the matrix $\mathbf{A}_t$ to evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \tau_t, \tag{9}$$

We assume the vector $[v_t', \eta_t', \tau_t', \tilde{v}_t']'$ to be distributed as

$$\begin{bmatrix} v_t \\ \eta_t \\ \tau_t \\ \tilde{v}_t \end{bmatrix} \mathrm{N} \sim (\mathbf{0}, \mathbf{V}), \; \text{with } \mathbf{V} = \begin{bmatrix} \mathbf{\Omega}_t & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \text{ and } \mathbf{G} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}. \tag{10}$$

The TVP-SVAR model can be written compactly as

$$Y_t = x_t'\tilde{B}_\mathbf{t} + A_{0,t}\varepsilon_t, \tag{11}$$

where $\mathbf{x_t} = \mathbf{I} \otimes [1, \mathbf{Y}_{t-1}, \mathbf{Y}_{t-2}, \ldots]$, $\tilde{\mathbf{B}}_\mathbf{t} = vec([c_t, \phi_{1,t}, \phi_{2,t}\ldots])$, $\mathrm{E}\left(\varepsilon_t\varepsilon_t'\right) = \mathbf{I}$, $\mathbf{A}_{0,t} = \mathbf{A}_t^{-1}\mathbf{H}_t^{1/2}\mathbf{P}$, where $\mathbf{P}$ is an orthonormal matrix ($\mathbf{PP} = \mathbf{I}$) that satisfies the zero-sign restrictions shown in Table 2.

This model is substantially more flexible than the one discussed in the previous Section. It is not only consistent with variation in the policy rule but also consistent with deviations from the rational expectations hypothesis. In this framework, agents do not

Table 2
Sign Restrictions – TVP-SVAR Model

Shocks\Variables	R_t	S_t	Y_t	π_t
Monetary policy shock	$\geq$	$\leq$	$\leq$	$\leq$
Spread shock	0	$\geq$	$\leq$	$\leq$
Demand shock	$\geq$	?	$\geq$	$\geq$
Supply shock	?	?	$\leq$	$\geq$

Notes. For variable definitions, see discussion of (5).

know the structural parameters and they use simple forecasting models to form their projections about future variables and, consequently, learn about the structure of the economy. This model seems very plausible during crisis periods where agents have no idea how shocks have changed the structure of the economy and they use simple 'rules of thumb' to learn about the new state. During the financial crisis, policy makers had to employ non-standard policy tools and, arguably, it makes sense for agents to 'abandon' the rational expectation hypothesis and use simple forecasting rules to learn about the structure of the economy, at least for a short period. If agents behave this way, we need to allow the parameters to vary over time as in the TVP-SVAR model to assess the effects of QE.

3.3.1. *Identification of structural shocks*
The shock identification scheme used for this model is identical to the one discussed in Section 3.2.1. The only difference is that two sets of restrictions have been dropped (those associated with the M4 and Stock Prices series) because the dimension of the VAR in this case has been reduced from six to four variables for reasons of tractability. Table 2 reports these restrictions.

4. Data

Our data set for the large BVAR comprises 43 variables, with monthly observations covering April 1993 to September 2010. UK variables include those capturing real activity, prices, money, the yield curve and financial markets.[10] Given that QE is expected to affect monetary aggregates and interest rates directly, the bulk of the domestic variables are interest rates, interest rate spreads and monetary aggregates. To incorporate potential international financial and economic linkages, we also include data for real activity, prices and the policy rates for the US and the euro area. We use log-levels for the variables except those which are already in growth rates. The list of variables are provided in Appendix Table A.1.

   The SVAR models were estimated using monthly and quarterly data on a smaller set of variables covering a longer period, from 1963–2011. The MS-SVAR uses monthly data, from February 1963 to March 2011, for the three-month Treasury bill rate, the 10-year government bond yield spread (defined as the 10-year government bond yield minus the three-month Treasury bill), annualised GDP growth, annualised CPI inflation, annualised M4 growth and the annual change in the FTSE All-Share index (stock prices). For the TVP-SVAR model, we use quarterly data, from 1968 Q1–2011 Q1,[11] for the three-month Treasury bill rate, the 10-year government bond yield spread (defined as the 10-year government bond yield minus the three-month Treasury bill rate), annualised GDP growth and annualised CPI inflation.

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[10] We obtain monthly GDP estimates from the National Institute of Economic and Social Research. These estimates are obtained from statistical projection and involve a fair amount of interpolation. Mitchell *et al.* (2005) discuss the methodology in detail.

[11] These data actually start from 1958 Q1 but we have used the first 10 years as a training sample to calibrate the priors.

5. Counterfactual Assumptions

Our counterfactual analysis is based on the empirical findings in Joyce *et al.* (2011), which suggest that QE may have depressed medium to long-term government bond yields (average 5 to 25-year spot rates) by about 100 basis points. We implement this impact on yields by changing the government bond spread, the spread between the relevant long-maturity bond yield in each model and the three-month Treasury bill rate. The resulting counterfactual simulations are conditional forecasts for real GDP and CPI inflation. We examine two scenarios: a policy scenario and a no policy scenario.

Under the policy scenario, which we describe as our baseline model prediction, we produce a counterfactual forecast taking the actual levels of long-term government bond spreads and Bank Rate that were observed from March 2009 to the end of our forecast horizon as our conditioning assumptions. We do not take the outturns for real GDP and CPI inflation as our baseline because the changes in these variables may also be due to changes in other factors that are not captured in the model. This means that we are only identifying the assumed impact of QE on the growth and inflation profiles, and disregarding all the other forces pushing up on demand. Consequently, the actual recovery may be higher than our model prediction, which does not capture these shocks.

For the no policy scenario, we assume that long-term government bond spreads would have been 100 basis points higher over the period from March 2009 onwards had QE not been implemented.[12] We also consider an alternative no policy scenario, where we adjust government bond spreads by 100 basis points and fix Bank Rate at 0.5%. We describe the two no policy scenarios as Bank Rate endogenous and Bank Rate exogenous.

To approximate the macroeconomic impact of QE, we compare the conditional forecasts for real GDP and CPI inflation under the policy scenario with those for the no policy scenario and take the difference between the two as our estimate. We are therefore using the change in the slope of the yield curve as our sole metric to determine the effects of QE on the macroeconomy. Lenza *et al.* (2010) and Baumeister and Benati (2010) use a similar approach to examine the effects of unconventional monetary policy on the macroeconomy. We do not examine the effects of other QE transmission channels. Implicit in our approach is the assumption that QE operates through expectations and that markets price in the total amount of asset purchases expected by the MPC. As noted, £75 billion of asset purchases were announced in March 2009, and this was extended to a total of £125 billion in May 2009, to a total of £175 billion in August 2009 and to a total of £200 billion in November 2009. However, evidence from event study analysis suggests that by far the largest reaction in gilt yields occured around March 2009 (Joyce *et al.*, 2011).

In the BVAR model, we focus on the 5 and 10-year government bond yield spreads to assess the potential macroeconomic effects of QE, so the adjustments for the no QE

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[12] This type of conditioning assumption is similar to the '*hard conditions*' discussed in Waggoner and Zha (1999).

counterfactual are applied to these spreads. For the smaller SVAR models, we apply the spread adjustment to the 10-year government bond spread.

6. Empirical Results

6.1. *Results from BVAR Model*

We set the lag order for our large BVAR model equal to one. For a model of this size, standard information criteria are difficult to use, so we rely on serial correlation tests on the residuals to arrive at the lag order. The residuals seemed to be well behaved, with little evidence of residual serial correlation. We set the tightness parameter following the approach used in Lenza *et al.* (2010). So the tightness parameter for the reported results ensures that the standard deviation of the residuals of the Bank Rate equation in the large BVAR is equivalent to those for the Bank Rate equation in a 'small' VAR. We choose 12 variables for the 'small' VAR, including both UK variables and foreign variables, to mimic the dynamics of a central bank monetary policy rule. The 'small VAR' is therefore used to pin down the prior for the BVAR.

We estimate the model using a rolling estimation approach and only use data from August 2007 to September 2010 to conduct our counterfactual analysis. For these simulations, we assume that under the two no policy scenarios, the 100-basis-point increase in long-term government bond yields (which is implemented through a rise in the 5 and 10-year gilt yield spread to the three-month Treasury bill rate) occurs in the initial period and yields remain at 100 basis points higher over the forecast horizon. We also conduct some sensitivity analysis by looking at the effects of a 80-basis-point and a 120-basis-point increase in spreads under the no policy scenario. We also vary the persistence of the QE shock by allowing the size of the adjustment on government bond spreads to vary over the forecast horizon.

Figure 2 illustrates the estimated effects of QE on real GDP and inflation using the Bank Rate endogenous scenario (*a*) and the Bank Rate exogenous scenario (*b*). As with any VAR model, the forecasts become less informative as the forecast horizon lengthens, as our focus is on the effects of QE, the counterfactual forecasts for GDP and CPI inflation are shown for the period that they lie below the baseline forecast which is typically around a year.

From the results, it appears that the decrease in long-term government bond spreads supported the level of real GDP during 2009 and prevented CPI inflation from becoming very low or negative. From Figure 2 (*a*), the Bank Rate endogenous scenario, leads to a maximum decrease of real GDP of about 0.7% in September 2009. In the Bank Rate exogenous scenario (Figure 2*b*), we observe a maximum fall of about 0.3% in the level real GDP in November 2009. The effects of QE on output are therefore more pronounced in the Bank Rate endogenous scenario compared with the Bank Rate exogenous scenario. This result is somewhat puzzling, as we would expect to see a larger effect in the case where the Bank Rate is fixed. This is perhaps a consequence of the fact that BVAR imposes little economic structure on the data. The effects on inflation are very similar across the two scenarios, however, with QE having a maximum effect of about 1% point on CPI inflation. The peak impact occurs in March 2010 for both scenarios. So our evidence for the effects of QE on real GDP would suggest that

(a) No Policy Counterfactual Simulations for the Level of
Real GDP (Bank Rate Endogenous)

2006 = 100

No QE Counterfactual (+100 bps)
No QE Counterfactual (+80 bps)
No QE Counterfactual (+120 bps)
No QE Counterfactual (Persistence)
Model Prediction
Real GDP

No Policy Counterfactual Simulations for Inflation
(Bank Rate Endogenous)

Per Cent

No QE Counterfactual (+100 bps)
No QE Counterfactual (+80 bps)
No QE Counterfactual (+120 bps)
No QE Counterfactual (Persistence)
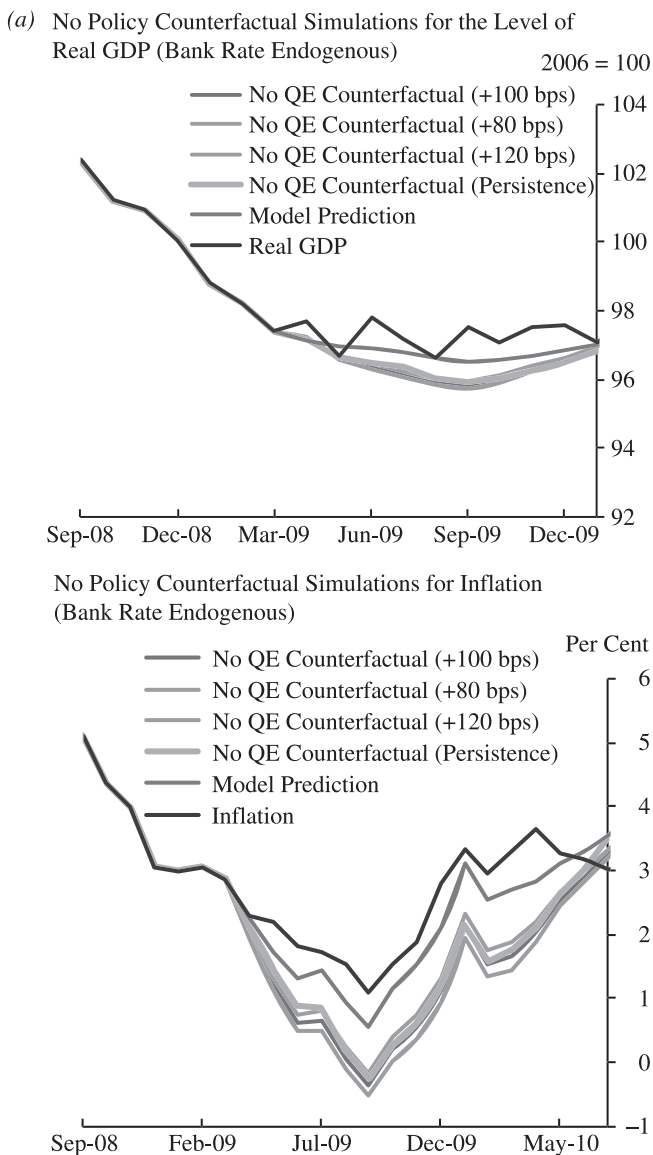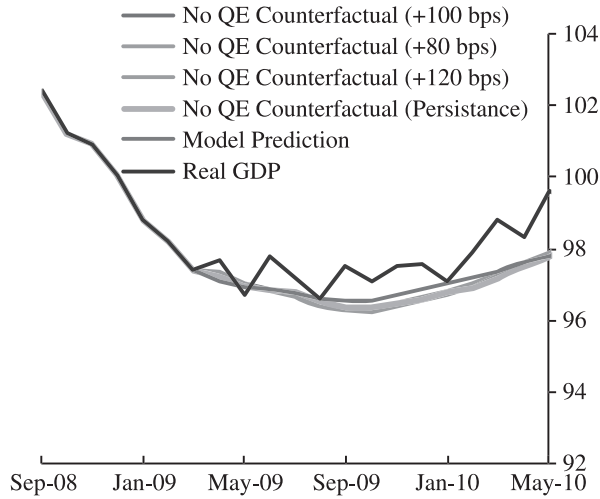Model Prediction
Inflation

Fig. 2. *BVAR Counterfactual Analysis*

the maximum effect occurs after about six to nine months, while the maximum effect
on inflation occurs after about a year.

   These are the maximum effects of QE. As noted previously, these are estimated by
comparing the no policy scenario with the policy scenario which is a forecast condi-
tional on the actual paths of the relevant interest rate spreads and the actual path for
Bank Rate over the forecast horizon. The effects would be larger if the counterfactual
were defined as the no policy scenario relative to the actual data, as the model
underpredicts output and inflation over the period.

(b) No Policy Counterfactual Simulations for the Level of
     Real GDP (Bank Rate Exogenous)

2006 = 100

— No QE Counterfactual (+100 bps)
— No QE Counterfactual (+80 bps)
— No QE Counterfactual (+120 bps)
— No QE Counterfactual (Persistence)
— Model Prediction
— Real GDP

No Policy Counterfactual Simulations for Inflation
(Bank Rate Exogenous)

Per Cent

— No QE Counterfactual (+100 bps)
— No QE Counterfactual (+80 bps)
— No QE Counterfactual (+120 bps)
— No QE Counterfactual (Persistance)
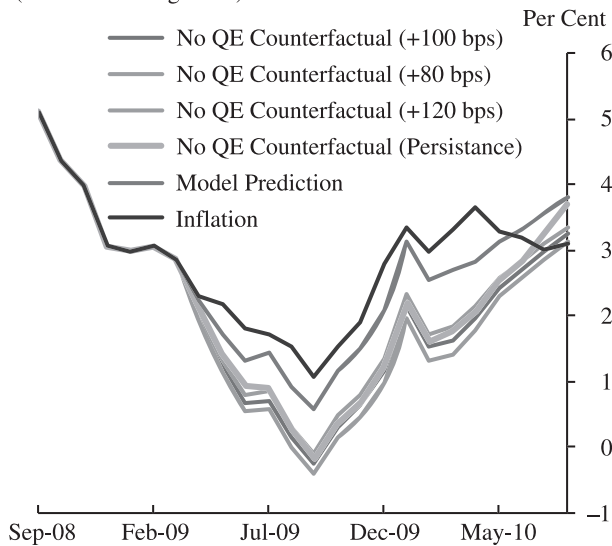— Model Prediction
— Inflation

Fig. 2. (Continued)

We also considered a number of other adjustments for the no policy scenario, as
shown in Table 3. This included examining the effects of larger and smaller changes in
spreads but, as the shock is linear in the spreads, the effects are simply proportional to
the 100-basis-point adjustment. To assess the persistence of the shock, we considered an
alternative adjustment profile for sensitivity analysis, which we call less persistence in
Figure 2. In this case, instead of assuming the effects on the government bond spread

Table 3

Maximum Effects of QE: Large BVAR

	CPI inflation		Real GDP level	
Adjustment\Estimate	BR endogenous	BR exogenous	BR endogenous	BR exogenous
80bp	0.82pp	0.85pp	0.61%	0.23%
100bp	1.03pp	1.07pp	0.72%	0.28%
120bp	1.24pp	1.28pp	0.83%	0.34%
Less persistence	0.96pp	0.94pp	0.65%	0.26%

Notes. BR is abbreviation for Bank Rate. The BR endogenous scenario is the forecast conditional on only the adjusted government bond spreads. The BR exogenous scenario is the forecast conditional on the adjusted government bond spreads and Bank Rate.

are constant, we assume that without QE there would have been a 60-basis-point increase in spreads in the first three months, a 100-basis-point increase for the next seven months and then a gradual decline of about 11-basis-points each month over the rest of the horizon.[13] Unsurprisingly this resulted in slightly smaller effects. But, overall, the results under these various alternative spread adjustment profiles were broadly similar to our central case.[14]

6.2. *Results from MS-SVAR Model*

For this model, the number of the regimes is decided before the model is estimated. This selection can be based on data-driven techniques – such as the marginal likelihood criterion (Chib, 1998) – or, as in this case, by prior knowledge. As noted previously, Figure 1 shows the estimated regime pattern. We listed four monetary policy regimes in the post-Bretton Woods era in Section 3.2. The first regime identified by the model ended in the early 1980s, which, although covering a period when different regimes may have operated, roughly coincides with the end of monetary targeting in the UK. The second regime ended in the early 1990s, a period when the UK left the Exchange Rate Mechanism and started inflation targeting. The end of the third regime is around the outset of the recent financial crisis.
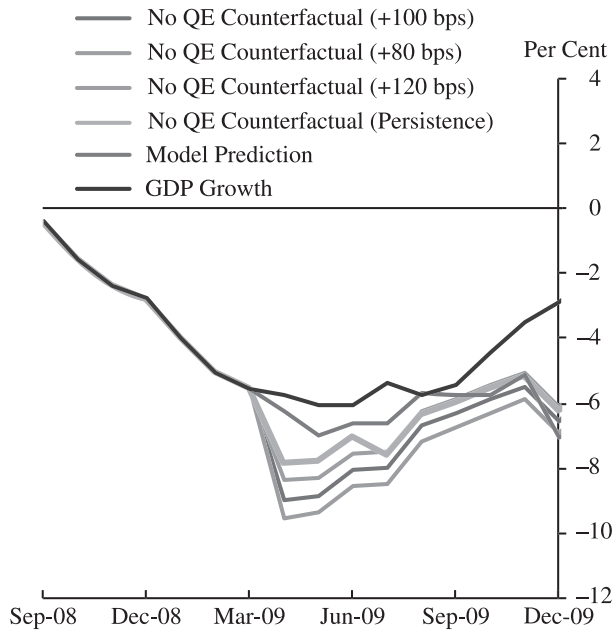
6.2.1. *Counterfactual scenario by imposing alternative spread paths*
This exercise is identical to the exercise in Section 6.1. Figure 3 plots the results from this experiment, where we focus on the effects of QE over the period until the no policy conditional forecasts return to the baseline. Similarly, the outcomes are grouped into two categories: Bank Rate endogenous and Bank Rate exogenous. The Bank Rate endogenous estimate (Figure 3*a*) is the forecast for output growth and CPI inflation

[13] In subsequent months after the seven months increase of 100 basis points, the increase in spreads will be equivalent to a decline of about 6% in the previous month's increase in spreads. For example, the increase in spreads in the 11th month is equivalent to 100 basis points minus $100(1/9)$. These types of conditioning assumptions are similar to the '*soft conditions*' described in Waggoner and Zha (1999).

[14] In addition, we also tried combining changes in yields with shocks to the money stock, with the aim of combining quantity and asset price effects. This is consistent with a standard portfolio balance approach; see Joyce *et al.* (2011). These results proved very sensitive to which monetary aggregate we assumed was affected by QE and they are not reported here. Further analysis of the effects of QE using a monetary approach is provided in another *Bank of England Working Paper* by Bridges and Thomas (2012).

(a) No Policy Counterfactual Simulations for GDP Growth
    (Bank Rate Endogenous)

——— No QE Counterfactual (+100 bps)
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
——— No QE Counterfactual (Persistence)
——— Model Prediction
——— GDP Growth

No Policy Counterfactual Simulations for Inflation
(Bank Rate Endogenous)

——— No QE Counterfactual (+100 bps)
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
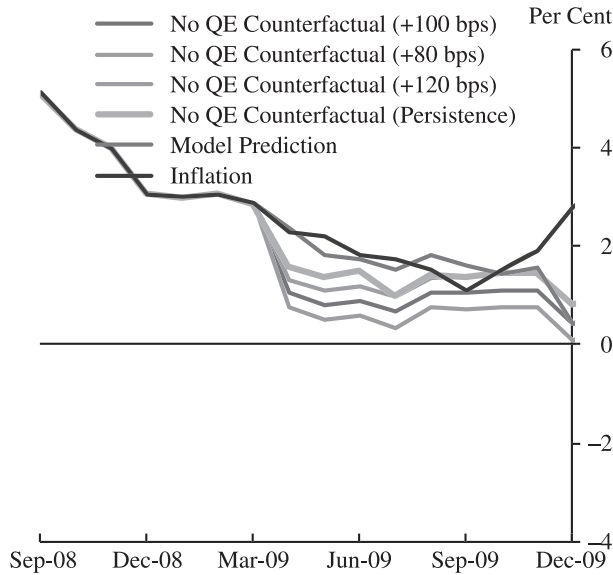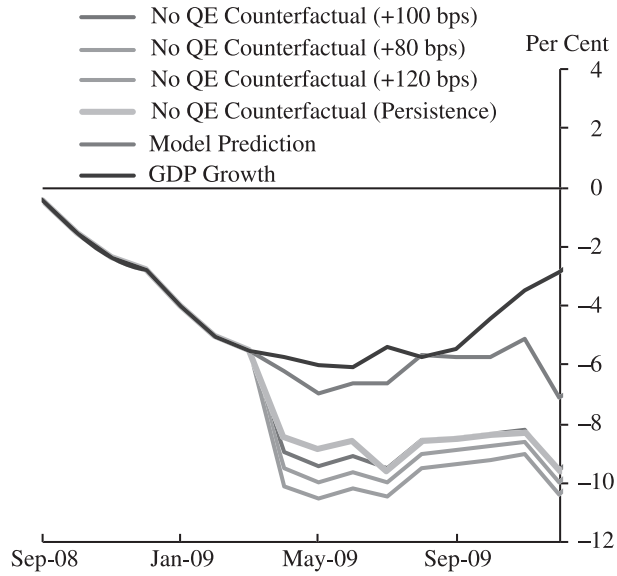——— No QE Counterfactual (Persistence)
——— Model Prediction
——— Inflation

Fig. 3. *Conditional Forecasts, MS-SVAR: GDP Growth and Inflation*

(b) No Policy Counterfactual Simulations for GDP Growth
(Bank Rate Exogenous)

——— No QE Counterfactual (+100 bps)   Per Cent
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
——— No QE Counterfactual (Persistence)
——— Model Prediction
——— GDP Growth

No Policy Counterfactual Simulations for Inflation
(Bank Rate Exogenous)

——— No QE Counterfactual (+100 bps)   Per Cent
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
——— No QE Counterfactual (persistence)
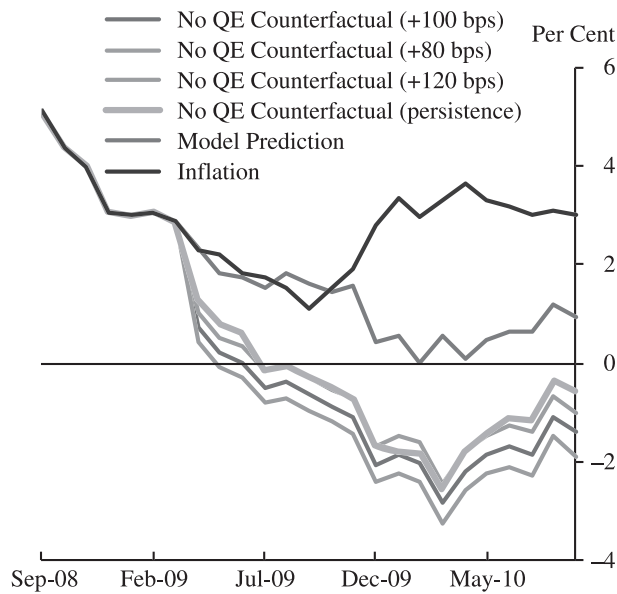——— Model Prediction
——— Inflation

Fig. 3. *(Continued)*

conditional on the adjusted government bond spread, under the no policy scenario. For the Bank Rate exogenous estimate, the forecast is conditional on the adjusted government bond spread and three-month Treasury bill rate (the proxy for Bank Rate in this model) fixed at 0.5% over the entire forecast period (Figure 3*b*).

Table 4 reports the peak effects on GDP growth and CPI inflation from these simulations. For a 100-basis-point contraction in spreads, the maximum impact on inflation and GDP growth occurs in April 2009 using the Bank Rate endogenous estimate and in March 2010 for the Bank Rate exogenous estimate. The large initial impact of the stimulus, using the Bank Rate endogenous estimate, occurs because in subsequent periods the unconstrained policy rate declines in response to lower inflation and output. The results for the less persistent case suggest that a staggered impact on spreads produces smaller effects compared with the standard case, where we assume there is a 100-basis-point increase in every period over the forecast horizon, especially for the case where Bank Rate is treated as endogenous. The MS-SVAR suggests that if we assumed the effects of QE on spreads were less persistent, the impact on output growth and CPI inflation would be smaller. These effects are more marked than in the case of the BVAR.

In the scenario where Bank Rate is exogenous, the effects on output and inflation are larger and inflation would have gone into negative territory. This means that without an additional policy instrument to affect expectations about demand, the economy would suffer from deflation and positive real interest rates which would depress demand even further (people expect inflation to fall, meaning consumption and investment goods are going to be cheaper in the future and, therefore, they postpone consumption and investment into the future). Uncertainty about states of the world is another mechanism that can squeeze demand. For instance, in periods of major financial crises, agents are particularly uncertain about the future, and to offset high-consumption variations (due to, say, the possibility of being unemployed) or/and to avoid big capital losses (from firms going bust), they increase their precautionary saving, which squeezes current demand.

6.2.2. *Impulse responses from MS-SVAR model*
The structural identification scheme allows us to examine the evolution of the variables in the different regimes following a particular shock. Focusing on the responses after a

Table 4
Maximum Effects of QE: MS-SVAR

Adjustment\Estimate	CPI inflation		GDP growth	
	BR endogenous	BR exogenous	BR endogenous	BR exogenous
80bp	1.03pp	3.01pp	2.14pp	3.68pp
100bp	1.31pp	3.38pp	2.72pp	4.08pp
120bp	1.59pp	3.79pp	3.29pp	4.45pp
Less persistence	0.75pp	3.11pp	1.57pp	3.96pp

Notes. BR is abbreviation for Bank Rate. The BR endogenous scenario is the forecast conditional on only the adjusted government bond spreads. The BR exogenous scenario is the forecast conditional on the adjusted government bond spreads and Bank Rate.
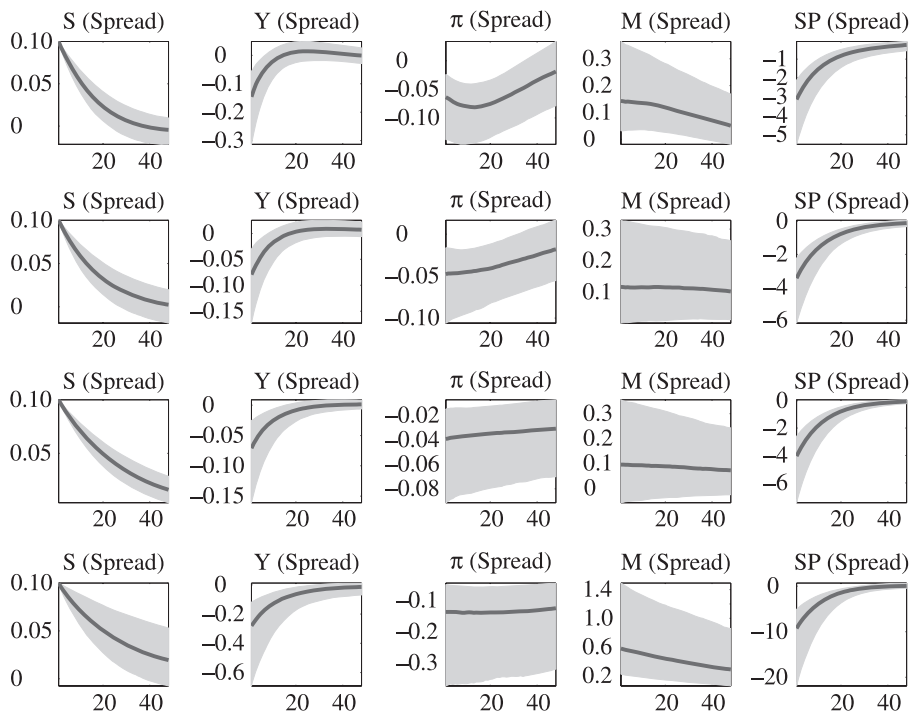
Fig. 4. *Responses After a Spread Shock in MS-SVAR*

shock to government bond spreads (the 'spread' shock), Figure 4 shows that this shock has significantly larger effects in the recent regime. For example, the effect of the shock (a 100-basis-point decrease in spreads) on GDP growth increases from about 0.6% in Regime 3 to about 2% in Regime 4.[15]

6.3. *Results from TVP-SVAR Model*

6.3.1. *Counterfactual scenario by imposing alternative spread paths*

Figure 5 and Table 5 show the results from the TVP-SVAR model. We focus on the effects of QE over the period until the no policy conditional forecasts return to the baseline. For a 100-basis-point contraction in spreads, the maximum effects on inflation and output growth occur in 2009 Q4 for the Bank Rate endogenous scenario and in 2010 Q1 for the Bank Rate exogenous scenario, i.e. three to four quarters after the start of QE. However, the model prediction is hugely pessimistic relative to the data out-turns. The poor forecasting power of the TVP-SVAR may be due to how agents learn about the evolution or structure of the economy. For example, if we consider the TVP-SVAR as the reduced-form version of a structural model where agents use simple forecasting rules to learn about the structure of the economy, then the results from

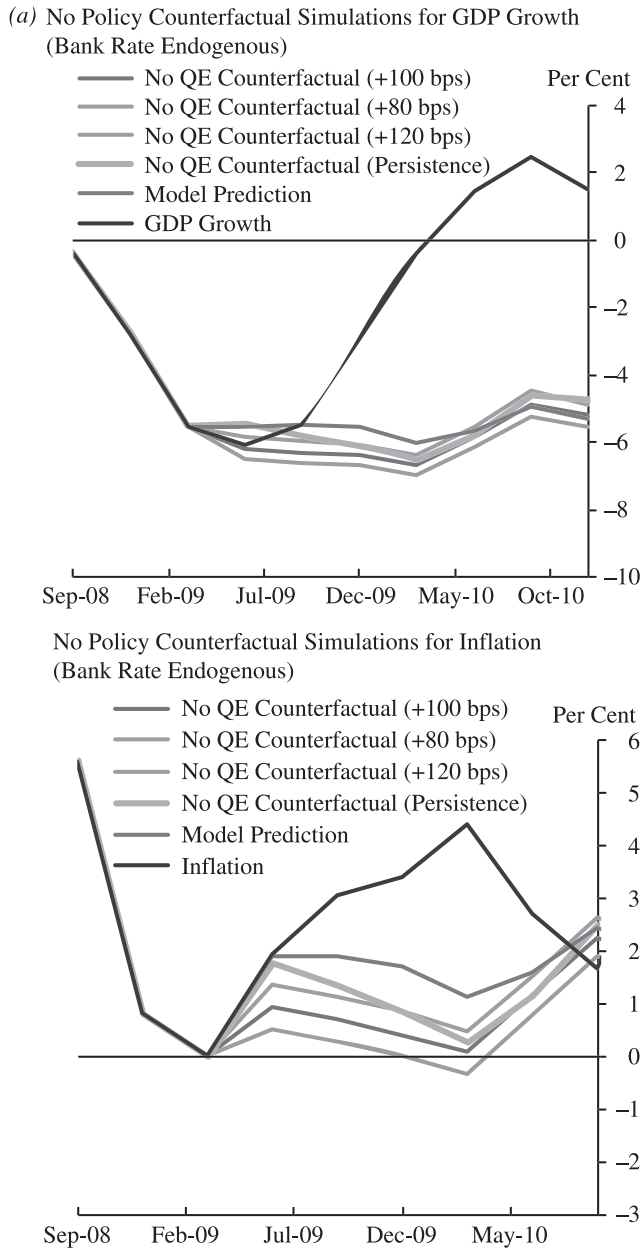[15] The responses have been normalised to make them comparable across different regimes.

(a) No Policy Counterfactual Simulations for GDP Growth
(Bank Rate Endogenous)

——— No QE Counterfactual (+100 bps)
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
——— No QE Counterfactual (Persistence)
——— Model Prediction
——— GDP Growth

No Policy Counterfactual Simulations for Inflation
(Bank Rate Endogenous)

——— No QE Counterfactual (+100 bps)
——— No QE Counterfactual (+80 bps)
——— No QE Counterfactual (+120 bps)
——— No QE Counterfactual (Persistence)
——— Model Prediction
——— Inflation

Fig. 5. *Conditional Forecasts, TVP-SVAR: GDP Growth and Inflation*

these conditional forecasts would suggest that the agents in this model learn very
slowly.[16] As the initial point of the forecast is in a downturn, agents seem to remain
pessimistic for a long period despite the stimulus from QE.

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[16] This is a standard feature of structural models with adaptive learning; see, for example, Evans and
Honkapohja (2001).

(b) No Policy Counterfactual Simulations for GDP Growth
(Bank Rate Exogenous)
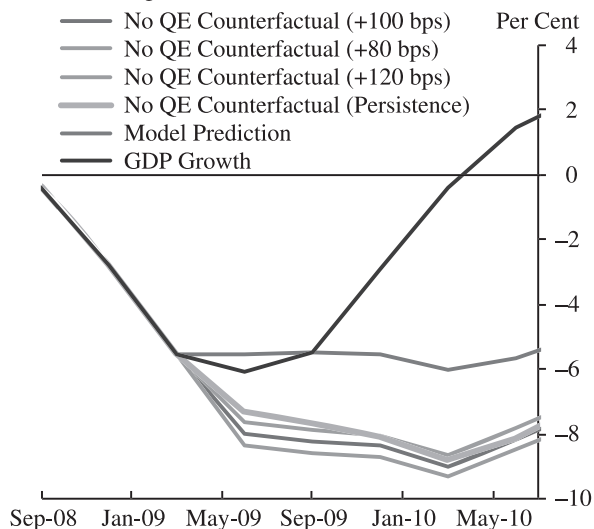
No Policy Counterfactual Simulations for Inflation
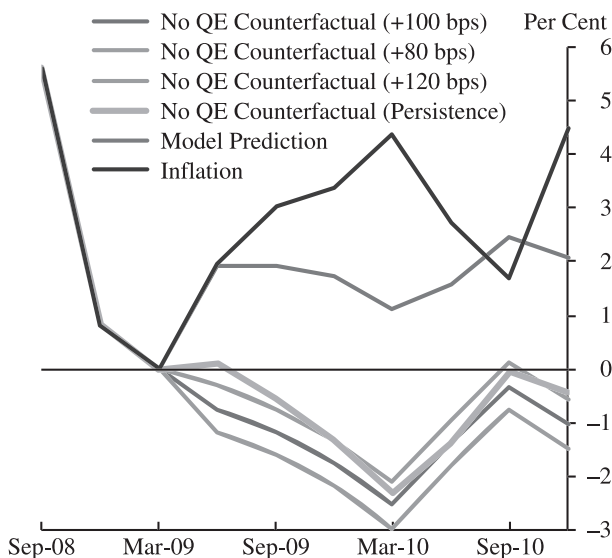(Bank Rate Exogenous)

Fig. 5. *(Continued)*

   The forecast performance not withstanding, we can still evaluate the effects of QE using
the TVP-SVAR model. Using the Bank Rate endogenous scenario, the effects of QE are
large and persistent reflecting agents' pessimism about the economic outlook. For the
Bank Rate exogenous scenario, the effects of QE are much larger. This is similar to the
MS-SVAR and may be due to the fact that when the policy rate is exogenised, recovery can

Table 5

Maximum Effects of QE: TVP-SVAR

Adjustment\Estimate	CPI inflation		GDP growth	
	BR endogenous	BR exogenous	BR endogenous	BR exogenous
80bp	0.87pp	3.20pp	0.56pp	2.64pp
100bp	1.30pp	3.63pp	0.86pp	2.98pp
120bp	1.71pp	4.09pp	1.15pp	3.29pp
Less persistence	0.87pp	3.42pp	0.56pp	2.82pp

Notes. BR is abbreviation for Bank Rate. The BR endogenous scenario is the forecast conditional on only the adjusted government bond spreads. The BR exogenous scenario is the forecast conditional on the adjusted government bond spreads and Bank Rate.

only be achieved by lowering spreads. In the case with less persistence of the QE shock, the TVP-SVAR suggests that the effects on output and CPI inflation would be commensurately smaller, broadly in line with findings for the MS-SVAR.

7. Summary of Empirical Results

Table 6 provides a summary of the key results from all three models employed in this article, showing the peak effects of a 100-basis-point QE shock from the counterfactual simulations. However, these effects occur at different times across the three models. Although the models are strictly not directly comparable due to their different dynamic structures and their informational content,[17] they jointly illustrate the range of potential macroeconomic impacts of QE.

The final row of the Table illustrates the effects implied by averaging across the models on the basis of an equally weighted probability for inflation and GDP.[18] The average model prediction indicates that, without the 100-basis-point contraction in government bond spreads following the implementation of QE, output would have been between 1.4% and 3.6% lower while CPI inflation would have been reduced by between 1.2% and 2.6% points relative to the baseline model prediction.

The range of these estimates reflects the different assumptions made about the policy rate in the two counterfactual simulations. Making Bank Rate exogenous produces a much larger range of results across the models. We put more emphasis on the more conservative model average results, the Bank Rate endogenous scenario. Our preferred average estimates from the three models therefore suggest that QE may have had a peak effect on the level of real GDP of around $1\frac{1}{2}\%$ and a peak effect on annual CPI inflation of about $1\frac{1}{4}\%$ points. Clearly, there is considerable uncertainty around all of the estimates, which increase as the forecast horizon lengthens. These results should be viewed as an attempt to quantify the macroeconomic effects of a 100-basis-point reduction in long-term interest rates. Of course, as noted earlier, QE may have affected the real economy in a variety of other ways and not just through its effect on long rates

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[17] Although from *Proposition 1* in Waggoner and Zha (1999) it is clear that the structural identification scheme plays no role in the conditional forecast, in this instance we could only loosely compare these models due to the different manner in which the reduced-form dynamics are modelled.

[18] To enable model comparison, we convert the GDP growth effects obtained from the smaller models to an equivalent GDP level effect.

Table 6

*Summary Table: Maximum Effect of QE (**100**-Basis-Point Contraction in Spreads)*

	CPI inflation		GDP level	
Adjustment\Estimate	BR endogenous	BR exogenous	BR endogenous	BR exogenous
BVAR	1.03pp	1.07pp	0.72%	0.28%
MS-SVAR	1.31pp	3.38pp	2.75%	5.13%
TVP-SVAR	1.30pp	3.63pp	0.86%	5.36%
Model average*	1.21pp	2.60pp	1.42%	3.59%

Notes. Model averaging done with an equally weighted probability. BR is abbreviation for Bank Rate. The BR endogenous scenario is the forecast conditional on only the adjusted government bond spreads. The BR exogenous scenario is the forecast conditional on the adjusted government bond spreads and Bank Rate.

(through confidence effects) but we have not investigated the impact of these other possible transmission channels in this article.

8. Some Caveats

We have noted a number of caveats to our analysis. We summarise these caveats in this Section and provide some suggestions for further research.

We do not identify how QE is transmitted into the economy, through either portfolio balance or/and inflation expectations or other channels. However, we believe that this does not alter our inference as we focus mainly on the reduced-form impact from a fall in spreads on output and inflation.[19] Although, the models used in this study are variants of VAR models that capture different aspects of the evolution in macro-economic variables, arguably, a structural DSGE-type model could potentially provide some further useful insights. However, as discussed previously, it is not immediately clear either what such a model would be or how well it would fit the data. Research on DSGE-type models for QE is ongoing. Our econometric models might be a useful way to take such models to the data.

Also, any form of counterfactual exercise is uncertain, particularly so in this case given the relative uniqueness of QE and the economic conditions in which it took place. We have also chosen to consider only models that account for structural change associated with parameter time-variation so as to minimise any biases arising from not accounting for structural change. A by-product of this choice is the likelihood that the estimates of the effects of QE are more uncertain than would otherwise be the case. However, as we have argued before, taking potential structural changes into account is extremely important for the validity of reduced-form modelling of QE.[20]

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[19] We have exclusively focused on the link between government bond spreads and macroeconomic variables, ignoring other possible channels. However, the consideration of additional channels would have complicated substantially the construction of the counterfactual exercise and rendered it potentially more controversial.

[20] A related issue is the possibility that the estimated time variation in the parameters may reflect the impact of policy changes rather than structural change. This may occur, for example, if the VAR models omit important variables; see, for example, Benati and Surico (2009), for arguments along this line. Note, however, that our findings on the impact of QE are relatively similar across three time varying parameter models that range from parsimonious to large scale. This provides some evidence that the estimated time-variation reflects underlying structural change.

Focusing our discussion on experiments where spreads fall by 100 basis points might seem restrictive. However, we do consider alternative scenarios, which deliver reasonably similar conclusions. A related issue is the possibility that QE may have influenced other variables in our models but we only condition on the spread and may miss contemporaneous shocks to those variables from QE. However, Pesaran and Smith (2012) present an autoregressive distributed lag (ARDL) model of GDP growth and bond spreads and undertake a counterfactual exercise which indicates that the effect of a 100-basis-point reduction in spreads has similar positive effects to GDP as those we find in the current article. As the model of Pesaran and Smith does not contain any other variables, there is no ambiguity as to the appropriate conditioning assumptions that define the counterfactual exercise. As a consequence, we consider the results of Pesaran and Smith (2012) to be a useful, relevant and positive robustness check on the results presented in this article.

Finally, the usual caveats concerning the econometric specification, estimation and validation of our models apply. However, our use of state of the art econometric techniques hopefully minimise the effects of such issues.

9. Conclusion

In this article, we provide new results on the potential macroeconomic effects of the Bank of England's QE programme during March 2009 to January 2010. We employ a multiple models approach, using three different time-series models. Results from a large BVAR model suggest that without QE real GDP would have fallen by even more during 2009 and inflation would have reached low or even negative levels. Results obtained from analysis using an MS-SVAR model and a TVP-SVAR model broadly support those obtained from the BVAR, if anything suggesting that the impact might have been even larger. Overall, our analysis would suggest that QE was an effective policy option during the financial crisis. However, the magnitude of its effects varies considerably across the different model specifications, and with the precise assumptions made to generate the counterfactual simulations, so these estimates are subject to considerable uncertainty.

We have qualified our analysis by a number of caveats. First, it is clear that any form of counterfactual exercise is very uncertain, particularly so in this case given the relative uniqueness of QE and the economic conditions in which it took place. Second, we have chosen a set of models that, while different, are nevertheless related and, therefore, may provide similar answers, which are subject to some uncertainty because they account for structural change associated with parameter time variation. Third, we have exclusively focused on the link between government bond spreads and macroeconomic variables, ignoring other possible transmission channels. Despite these caveats, our multiple time-series approach provide a useful benchmark for assessing the macroeconomic impact of QE.

A. Appendix

A.1. *Estimation of Large BVAR Model*

In what follows, we briefly discuss the estimation of the large BVAR described in (1). We can compactly rewrite the VAR as follows

$$\mathbf{Y} = \mathbf{X}_h \mathbf{\Psi}_h + \mathbf{E}, \tag{A.1}$$

where $\mathbf{Y} = [y_{h+1}, \ldots, y_T]'$ is a $T \times N$ matrix containing all the data points in y_t, $\mathbf{X}_h = [1 \ \mathbf{Y}_{-h}]$ is a $T \times M$ matrix containing a vector of ones (1) in the first columns and the h-th lag of $\mathbf{Y}$ in the remaining columns, $\mathbf{\Psi}_h = [\Phi_{0,h} \Phi_{1,h}]'$ is a $M \times N$ matrix, and $\mathbf{E} = [\varepsilon_{h+1}, \ldots, \varepsilon_T]'$ is a $T \times N$ matrix of disturbances. As only one lag is considered, we have $M = N + 1$. The prior distribution can then be written as follows

$$\mathbf{\Psi}_h | \mathbf{\Sigma} \sim \mathrm{N}(\mathbf{\Psi}_0, \mathbf{\Sigma} \otimes \mathbf{\Omega}_0), \mathbf{\Sigma} \sim \mathrm{iW}(v_0, S_0). \tag{A.2}$$

Note that $\mathbf{\Psi}_h \mid \mathbf{\Sigma}$ is a matric-variate normal distribution where the prior expectation $\mathrm{E}[\mathbf{\Psi}_h] = \mathbf{\Psi}_0$ and prior variance $\mathrm{Var}[\mathbf{\Psi}_h] = \mathbf{\Sigma} \otimes \mathbf{\Omega}_0$ are set according to (2). The prior variance matrix has a Kroneker structure $\mathrm{Var}[\mathbf{\Psi}_h] = \mathbf{\Sigma} \otimes \mathbf{\Omega}_0$ where $\mathbf{\Sigma}$ is the variance matrix of the disturbances and the elements of $\mathbf{\Omega}_0$ are given by $Var[\Phi_{1,h}^{(ij)}]$ in (2). Since the normal-inverted Wishart prior is conjugate, the conditional posterior distribution of this model is also normal-inverted Wishart (Zellner, 1973):

$$\mathbf{\Psi}_h | \mathbf{\Sigma}, \mathbf{Y} \sim N(\bar{\mathbf{\Psi}}, \mathbf{\Sigma} \otimes \bar{\mathbf{\Omega}}), \mathbf{\Sigma} | \mathbf{Y} \sim \mathrm{iW}(\bar{\mathbf{v}}, \bar{\mathbf{S}}), \tag{A.3}$$

where the bar denotes that the parameters are those of the posterior distribution. Defining $\hat{\mathbf{\Psi}}$ and $\hat{\mathbf{E}}$ as the OLS estimates, we have that $\bar{\mathbf{\Psi}} = (\mathbf{\Omega}_0^{-1} + \mathbf{X}'\mathbf{X})^{-1}(\mathbf{\Omega}_0^{-1}\mathbf{\Psi}_0 + \mathbf{X}'\mathbf{Y})$, $\bar{\mathbf{\Omega}} = (\mathbf{\Omega}_0^{-1} + \mathbf{X}'\mathbf{X})^{-1}$, $\bar{\mathbf{v}} = v_0 + T$, and $\bar{\mathbf{S}} = \hat{\mathbf{\Psi}}'\mathbf{X}'\mathbf{X}\hat{\mathbf{\Psi}} + \mathbf{\Psi}_0'\mathbf{\Omega}_0^{-1}\mathbf{\Psi}_0 + \mathbf{\Psi}_0 + \hat{\mathbf{E}}'\hat{\mathbf{E}} - \hat{\mathbf{\Psi}}'\bar{\mathbf{\Omega}}^{-1}\hat{\mathbf{\Psi}}$.

To perform inference and forecasting, one needs the full joint posterior distribution and the marginal distributions of the parameters $\bar{\mathbf{\Psi}}$ and $\mathbf{\Sigma}$. One could use the conditional posteriors in (A.3) as a basis of a Gibbs sampling algorithm that drawing in turn from the conditionals $\mathbf{\Psi}_h \mid \mathbf{\Sigma}, \mathbf{Y}$ and $\mathbf{\Sigma} \mid \mathbf{Y}$ would eventually produce a sequence of draws from the joint posterior $\mathbf{\Psi}_h \mathbf{\Sigma} \mid \mathbf{Y}$ and the marginal posteriors $\mathbf{\Psi}_h \mid \mathbf{Y}, \mathbf{\Sigma} \mid \mathbf{Y}$, as well as the posterior distribution of any function of these coefficients (e.g. multi-step forecasts or impulse responses).

Still, if one is interested only in the posterior distribution of $\mathbf{\Psi}_h$ (rather than in any non-linear function of it) there is an alternative to simulation: by integrating out $\mathbf{\Sigma}$ from (A.3). Zellner, 1973), has shown that the marginal posterior distribution of $\mathbf{\Psi}_h$ is a matric-variate t:

$$\mathbf{\Psi}_h | \mathbf{Y} \sim \mathrm{MT}(\bar{\mathbf{\Psi}}, \bar{\mathbf{\Omega}}^{-1}, \bar{\mathbf{S}}, \bar{\mathbf{v}}). \tag{A.4}$$

The expected value for this distribution is given by

$$\bar{\mathbf{\Psi}} = (\mathbf{\Omega}_0^{-1} + \mathbf{X}'\mathbf{X})^{-1}(\mathbf{\Omega}_0^{-1}\mathbf{\Psi}_0 + \mathbf{X}'\mathbf{Y}), \tag{A.5}$$

which is obviously extremely fast to compute. Recalling that $\hat{\mathbf{\Psi}}$ is the OLS estimator, and using the normal equations $(\mathbf{X}'\mathbf{X})^{-1}\hat{\mathbf{\Psi}} = \mathbf{X}'\mathbf{Y}$ we can rewrite this as follows

$$\bar{\mathbf{\Psi}} = (\mathbf{\Omega}_0^{-1} + \mathbf{X}'\mathbf{X})^{-1}(\mathbf{\Omega}_0^{-1}\mathbf{\Psi}_0 + \mathbf{X}'\mathbf{X}\hat{\mathbf{\Psi}}), \tag{A.6}$$

which shows that the posterior mean of $\mathbf{\Psi}_h$ is a weighted average of the OLS estimator and of the prior mean $\mathbf{\Psi}_0$, with weights proportional to the inverse of their respective variances. In the presence of a tight prior (i.e. when $\theta \to 0$), the posterior estimate will collapse to $\bar{\mathbf{\Psi}} = \mathbf{\Psi}_0$, while with a diffuse prior (i.e. when $\theta \to \infty$) the posterior estimate will collapse to the unrestricted OLS estimate.

Given the posterior mean $\bar{\mathbf{\Psi}} = [\bar{\mathbf{\Phi}}_{0,h} \bar{\mathbf{\Phi}}_{1,h}]'$, it is straightforward to produce forecasts up to h steps ahead simply by setting:

$$\hat{y}_{t+h} = \bar{\mathbf{\Phi}}_{0,h} + \bar{\mathbf{\Phi}}_{1,h}y_t. \tag{A.7}$$

As shown by Banbura *et al.* (2010), it is also possible to implement the prior described above using a set of dummy observations. Consider adding T_d dummy observations $\mathbf{Y_d}$ and $\mathbf{X_d}$ such that their moments coincide with the prior moments: $\mathbf{\Psi}_0 = (\mathbf{X}_d'\mathbf{X}_d)^{-1}\mathbf{X}_d'\mathbf{Y}_d$, $\mathbf{\Omega}_0 = (\mathbf{X}_d'\mathbf{X}_d)^{-1}$, $\mathbf{v}_0 = \mathbf{T}_d - M - N - 1$, $\mathbf{S}_0 = (\mathbf{Y}_d - \mathbf{X}_d\mathbf{\Psi}_0)'(\mathbf{Y}_d - \mathbf{X}_d\mathbf{\Psi}_0)$. Augmenting the system in 12 with the dummy observations gives

$$\mathbf{Y}^+ = \mathbf{X}_h^+ \mathbf{\Psi}_h + \mathbf{E}^+, \tag{A.8}$$

where $\mathbf{Y}^+ = (\mathbf{Y}'\mathbf{Y}'_d)'$ and $\mathbf{E}^+ = (\mathbf{E}'\mathbf{E}'_d)'$ are $(T + T_d) \times N$ matrices and $\mathbf{X}^+ = (\mathbf{X}'\mathbf{X}'_d)'$ is a $(T + T_d) \times M$ matrix. Then, it is possible to show that the OLS estimator of the augmented system (given by the usual formula $(\mathbf{X}_h^{+\prime}\mathbf{X}_h^+)^{-1}\mathbf{X}_h^{+\prime}\mathbf{Y}^+$) is numerically equivalent to the posterior mean $\bar{\boldsymbol{\Psi}}$.

A.1.1. *Prior tightness*

To make the prior operational, one needs to choose the value of the hyperparameter ϕ. We discuss a number of methods for addressing this issue. The marginal data density of the model can be obtained by integrating out all the coefficients, i.e. defining $\boldsymbol{\Theta}$ as the set of all the coefficients of the model, the marginal data density is

$$p(\mathbf{Y}) = \int p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})\mathrm{d}\boldsymbol{\Theta}. \tag{A.9}$$

Under our normal-inverted Wishart prior the density $p(\mathbf{Y})$ can be computed in closed form (Bauwens *et al.*, 1999). At each point in time ϕ could be chosen by maximising:

$$\phi_t^* = \arg\max_\phi \ln p(\mathbf{Y}). \tag{A.10}$$

This method has been used by Carriero *et al.* (2010). However, as discussed there, such a method may have a tendency to choose low values for the tightness parameter implying a large weight on the prior. It is important for our purposes to give considerable weight on the data. We therefore adopt an alternative approach whereby the tightness parameter is chosen by matching the fit of particular equations in the large VAR to those from smaller VAR models. Lenza *et al.* (2010) use a similar approach to set tightness. We find this approach produces a reasonable balance between the effects of priors and data that is appropriate for our analysis.

A.2. *Estimation of MS-SVAR Model*

We follow Chib (1998) and adopt a Bayesian Gibbs sampling approach to the estimation of the MS-SVAR models. Here, we briefly describe the main steps of the algorithm. The Gibbs sampling algorithm proceeds in the following steps:

(i) *Sampling*: $\mathbf{s}_t$: Following Kim and Nelson (1999), we use the Multi-Move Gibbs sampling algorithm to draw s_t from the joint conditional density $f(\mathbf{s}_t | \mathbf{Y}_t, \mathbf{c}_S, \mathbf{B}_{1,S}, \ldots, \mathbf{B}_{K,S}, \boldsymbol{\Omega}_S, \tilde{\mathbf{P}})$. We impose the restriction that each regime must have at least $N \times (K + 1) + 2$ observations, where N denotes the number of endogenous variables in the VAR, to ensure sufficient degrees of freedom for each regime.

(ii) *Sampling*: $\mathbf{c}_S, \mathbf{B}_{1,S}, \ldots, \mathbf{B}_{K,S}, \boldsymbol{\Omega}_S$: Conditional on a draw for S_t, the model is simply a sequence of Bayesian VAR models. The regime-specific VAR coefficients are sampled from a normal distribution and the covariances are drawn from an inverted Wishart distribution. For the first M regimes, we use a normal inverse Wishart prior (Kadiyala and Karlsson, 1997). However, as described in detail below, we employ a (normal diffuse) prior distribution for the VAR coefficients of the final regime, which is compatible with the identification of the shock to the government bond spread. In our sample, the recent financial crisis coincides with the final regime of the estimated VAR model. The prior on the VAR coefficients in this regime implies that the policy rate does not respond to lagged changes in other endogenous variables. This assumption is compatible with restrictions used to identify the shock to the bond yield spread and reflects the fact that the policy rate is close to the ZLB.

(iii) *Sampling*: $\tilde{\mathbf{P}}$: Given the state variables $\mathbf{s}_t$, the non-zero elements of the transition probability matrix are independent of $\mathbf{Y}_t$ and the other parameters of the model and are drawn from a Dirichlet posterior.

We estimate the MS-SVAR model using 2,00,000 replications of the Gibbs sampler and discard the first 1,90,000 as burn-in. Figure A1 plots the 20th-order autocorrelation for the key parameters of the benchmark model. These are close to zero providing evidence in favour of convergence.

A.3. *Estimation of TVP-SVAR Model*

The TVP-SVAR model is estimated using the Bayesian methods described in Kim and Nelson (1999). In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. Here, we summarise the basic algorithm which involves the following steps:

(*i*) The VAR coefficients $\tilde{\mathbf{B}}_t$ and the off-diagonal elements of the covariance matrix $\mathbf{A}_t$ are simulated by using the methods described in Carter and Kohn (1994). As is common practice in this literature (Cogleya and Sargent, 2005) we impose the constraint that $\tilde{\mathbf{B}}_t$ should be stable at each point in time.

(*ii*) The volatilities of the reduced-form shocks $\mathbf{H}_t$ are drawn using the date-by-date blocking scheme introduced in Jacquier *et al.* (2004).

(*iii*) The hyperparameters $\mathbf{Q}$ and $\mathbf{S}$ are drawn from an inverse Wishart distribution while the elements of $\mathbf{G}$ are simulated from an inverse gamma distribution.

Figure A2 shows the recursive means of the retained draws appear stable, providing evidence of convergence.

The lag length is set at two. The data sample runs from 1958 Q1–2011 Q1 and we use the first 10 years of data as a training sample that is used to calibrate priors.

A.4. *Data Appendix for Large BVAR Model*

The data set for the large BVAR model is given in Table A1.

Table A1

Data Appendix for Large BVAR Model

No.	Variable	Source	No.	Variable	Source
1	US industrial production	DS	23	20-year UK gilts	BofE
2	US CPI	DS	24	15-year UK gilts	BofE
3	Euro-area industrial production	DS	25	7-year UK gilts	BofE
4	Euro-area HICP	ECB	26	3-year UK gilts	BofE
5	UK GDP	NIESR	27	5-year 5-year implied inflation	BofE
6	UK industrial production	ONS	28	6-month Libor	BG
7	Brent dollar oil price	DS	29	12-month Libor	BG
8	UK CPI	ONS	30	FTSE All-Share index	DS
9	UK-PPI	ONS	31	FTSE All-Share dividend yield	DS
10	UK-UEMP	ONS	32	FTSE All-Share price-earnings ratio	DS
11	UK house price index	HF	33	UK exchange rate index	BofE
12	10-year gilt – T-bill spread	BofE	34	US dollar-sterling exchange rate	BofE
13	UK consumer confidence	EC	35	Euro-sterling exchange rate	BofE
14	M4	BofE	36	T-bill – Bank Rate spread	BofE
15	M3	BofE	37	3-month Libor – T-bill spread	BofE
16	Retail deposit and cash in M4	BofE	38	3-month Libor-Bank Rate spread	BofE/BG
17	Secured lending to individuals	BofE	39	2-year gilt – T-bill spread	BofE
18	M4 net lending to private sector	BofE	40	5-year gilt – T-bill spread	BofE
19	M4 lending	BofE	41	Bank Rate	BofE
20	Household M4	BofE	42	US federal funds rate	Fed
21	PNFC M4	BofE	43	Euro-MRO interest rate	BD/BG
22	OFC-M4	BofE			

Data Sources: Bank of England (BofE), Board of Governors of the Federal Reserve System (Fed), Bundesbank (BD), European Central Bank (ECB), European Commission (EC), National Institute of Economic and Social Research (NIESR), Datastream (DS), Bloomberg (BG), Office for National Statistics (ONS) and Halifax (HF). Data transformation: We use log-levels for the variables except those which are already in growth rates.
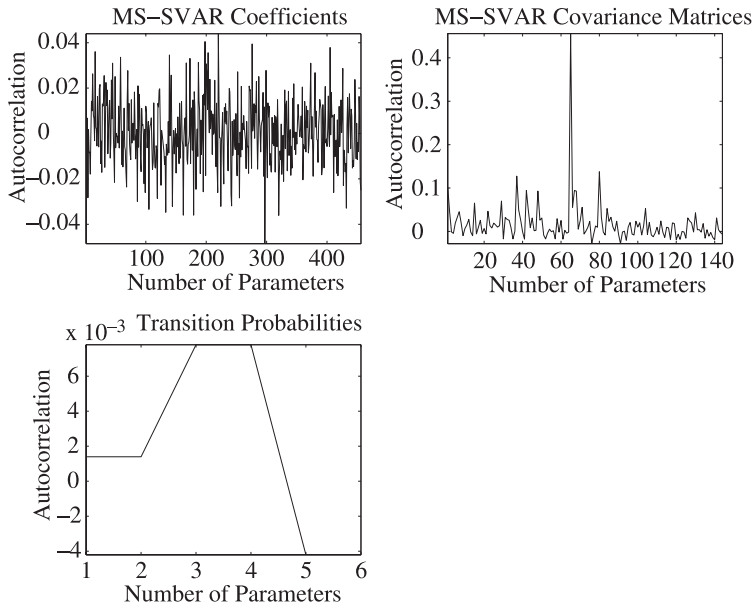
A.5. *Charts*

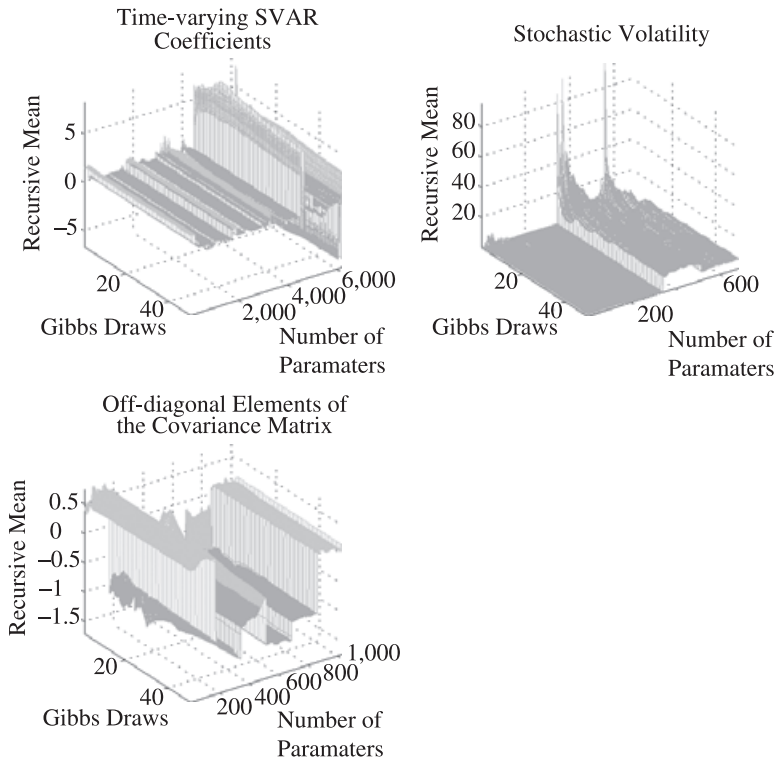Fig. A1. *Convergence Diagnostic Statistics: MS-SVAR*

Fig. A2. *Convergence Diagnostic Statistics: TVP-SVAR*

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